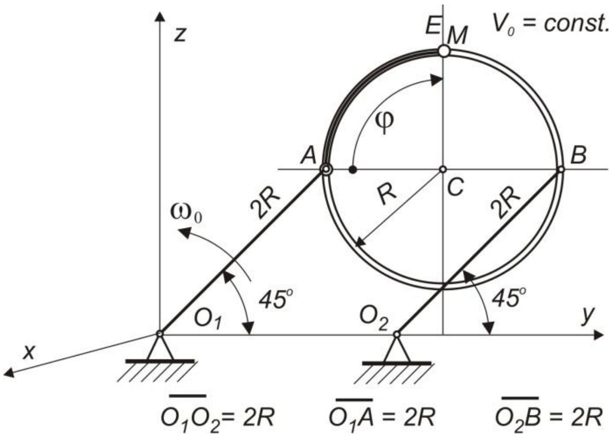
Zadatak 4.

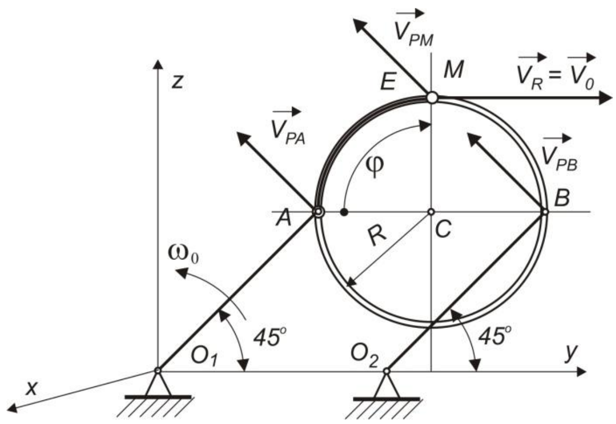
Kroz cev kružnog oblika, poluprečnika R, kreće se tačka M relativnom brzinom konstantnog intenziteta $V_R = V_0$. Cev se kreće posredstvom poluga $O_1A=2R$ i $O_2B=2R$, koje se obrću oko nepomičnih osa O_1 i O_2. Poluga O_1A se obrće ugaonom brzinom konstantnog intenziteta $\omega_{OA} = \omega_0 = const.$ U trenutku kada poluga O_1A gradi ugao od 45° u odnosu na horizontalu O_1O_2 tačka M je u najvišoj tački E. Odrediti apsolutnu brzinu i ubrzanje tačke M u tom trenutku.



Rešenje:

Kretanje tačke M po cevi je relativno kretanje, kako je naglašeno. Brzina tačke M ima pravac tangente na putanju, pa je u trenutku kada je tačka M u položaju E tangenta horizontalna, odnosno u pravcu y ose.

$$\vec{V}_R = V_0 \vec{j} = const.$$



Obruč se kreće translatorno - krivolinijska translacija je prenosno kretanje pa su brzine svih tačaka cevi su jednake.

$$V_A = V_B = V_E = 2R\omega_0$$

$$\vec{V}_P = -2R\omega_0 \frac{\sqrt{2}}{2} \vec{j} + 2R\omega_0 \frac{\sqrt{2}}{2} \vec{k}$$

Apsolutna brzina tačke M kada dospe u položaj E

$$\vec{V} = \vec{V}_P + \vec{V}_R = \left(V_0 - \sqrt{2}R\omega_0\right) \vec{j} + \sqrt{2}R\omega_0 \vec{k}$$

$$\vec{V} = \sqrt{\left(V_0 - \sqrt{2}R\omega_0\right)^2 + \left(\sqrt{2}R\omega_0\right)^2}$$

$$\vec{V} = \sqrt{2R^2{\omega_0}^2 + V_0^2 - 2\sqrt{2}V_0R\omega_0 + 2R^2{\omega_0}^2} =$$

$$\vec{V} = \sqrt{4R^2{\omega_0}^2 + V_0^2 - 2\sqrt{2}V_0R\omega_0}$$

Apsolutno ubzanje tačke M kada dospe u položaj E

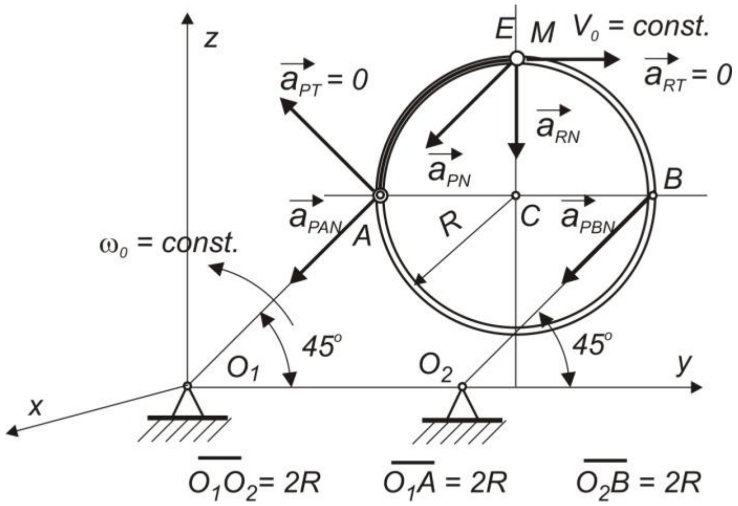
$$\vec{a} = \vec{a}_P + \vec{a}_R + \vec{a}_C$$

Prenosno ubzanje

$$\vec{a}_P = \vec{a}_{PA} = \vec{a}_{PT} + \vec{a}_{PN}$$

$$a_{PT} = 2R\frac{d\omega_0}{dt} = 0$$

$$a_{PN} = 2R\omega_0^2$$



$$\vec{a}_P = \vec{a}_{PN} = -\sqrt{2}R\omega_0^2\,\vec{j} - \sqrt{2}R\omega_0^2\,\vec{k}$$

Relativno ubrzanje

$$\vec{a}_R = \vec{a}_M = \vec{a}_{RT} + \vec{a}_{RN}$$

$$a_{RT} = \frac{dV_0}{dt} = 0$$

$$a_{RN} = \frac{V_R^2}{R} = \frac{V_0^2}{R} \qquad \vec{a}_R = \frac{V_R^2}{R}\,\vec{k}$$

Koriolisovo ubrzanje - translatorno prenosno kretanje pa nema ugaone prenosne brzine i Koriolisovo ubrzanje je jednako nuli.

$$\vec{a}_C = 2\vec{\omega}_P \times \vec{V}_R = 0 \times \vec{V}_R = 0$$

Apsolutno ubzanje tačke M kada dospe u položaj E

$$\vec{a} = \vec{a}_P + \vec{a}_R = -\sqrt{2}R\omega_0^2\,\vec{j} - \left(\sqrt{2}R\omega_0^2 + \frac{V_R^2}{R}\right)\vec{k}$$

$$a = \sqrt{\left(\sqrt{2}R\omega_0^2\right)^2 + \left(\sqrt{2}R\omega_0^2 + \frac{V_R^2}{R}\right)^2}$$

$$a = \sqrt{2R^2\omega_0^4 + 2R^2\omega_0^4 + 2\sqrt{2}V_R^2\omega_0^2 + \frac{V_R^4}{R^2}}$$

$$a = \sqrt{4R^2\omega_0^4 + \frac{V_R^4}{R^2} + 2\sqrt{2}V_R^2\omega_0^2}$$

Apsolutna brzina tačke M kada dospe u položaj tačke H odnosno $T = \frac{\sqrt{5}}{2k}$

$$\vec{V}_M = \vec{V}_P + \vec{V}_R$$

$$V_{Mx} = -V_P + V_R cos\alpha = -\sqrt{5}\ \pi Lk + 8\sqrt{5}\ Lk = \sqrt{5}\ Lk(8-\pi)$$

$$V_{My} = V_R sin\alpha = 4\sqrt{5}\ Lk$$

$$\vec{V}_M = \sqrt{5}\ Lk(8-\pi)\ \vec{i} + 4\sqrt{5}\ Lk\ \vec{j}$$

Apsolutno ubrzanje tačke M kada dospe u položaj tačke H odnosno $T = \frac{\sqrt{5}}{2k}$

$$\vec{a}_M = \vec{a}_P + \vec{a}_R + \vec{a}_C = \vec{a}_P + \vec{a}_R + 0$$

$$\vec{a}_P = \vec{a}_H = \vec{a}_A = -2L\pi k^2\vec{i} - 5L\pi k^2\vec{j}$$

$$\vec{a}_R = 16\ Lk^2\ \vec{i} + 8\ Lk^2\ \vec{j}$$

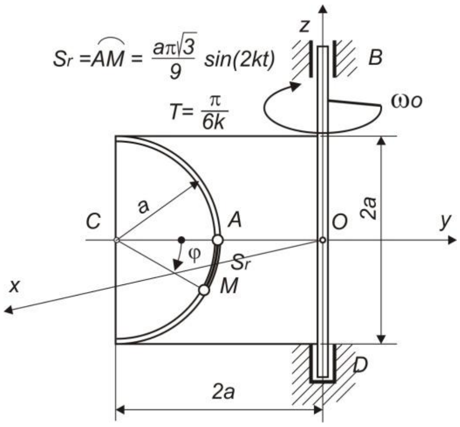
$$\vec{a}_C = 0$$

$$\vec{a}_M = -2L\pi k^2\vec{i} - 5L\pi k^2\vec{j}\ + 16\ Lk^2\ \vec{i} + 8\ Lk^2\ \vec{j} = 14\ Lk^2\ \vec{i} + 3\ Lk^2\ \vec{j}$$

$$\vec{a}_M = \sqrt{(14\ Lk^2)^2 + (3\ Lk^2)^2} = 14.3178\ Lk^2$$

Zadatak 5. ISPITNI

Kvadratna ploča strane 2a se obrće oko vertikalne ose konstantnom ugaonom brzinom $\omega_0 = const.$ Po kružnom žljebu poluprečnika a se kreće tačka po zakonu



$s_R = \frac{a\pi\sqrt{3}}{9} sin(2kt).$ Gde je k konstanta.

Odrediti apsolutnu brzinu i ubrzanje tačke M u trenutku $T = \frac{\pi}{6k}$.

Rešenje:

Kretanje tačke M po žljebu je relativno kretanje. Brzina tačke M ima pravac tangente na putanju, U trenutku $T = \frac{\pi}{6k}$*. tačka M je pod uglom* φ_1 *.*

Dat je zakon kretanja u prirodnim koordinatama odakle se dobija

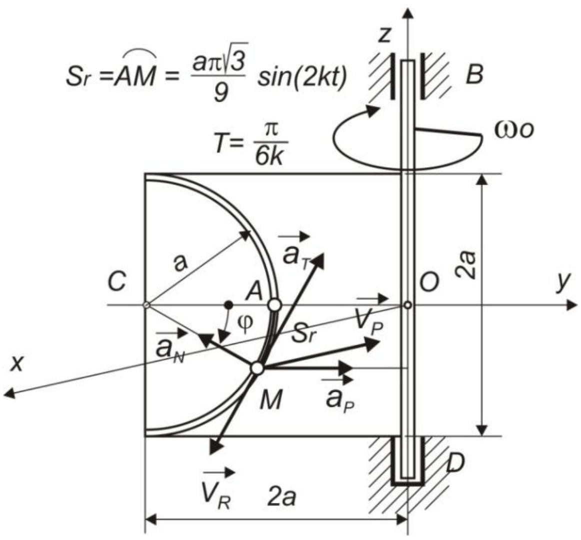
$$s_R = \frac{a\pi\sqrt{3}}{9} sin(2kt)$$

$$s_R = a \cdot \varphi = \frac{a\pi\sqrt{3}}{9} sin(2kt) \to \varphi = \frac{s_R}{a} = \frac{a\pi\sqrt{3}}{9\,a} sin(2kt) = \frac{\pi\sqrt{3}}{9} sin(2kt)$$

U trenutku $T = \frac{\pi}{6k}$

$$s_{R1} = \frac{a\pi\sqrt{3}}{9} sin(2kt) = \frac{a\pi\sqrt{3}}{9} sin\left(2k\frac{\pi}{6k}\right) = \frac{a\,\pi}{6}$$

$$\varphi_1 = \frac{\pi\sqrt{3}}{9} sin(2kt) = \frac{\pi\sqrt{3}}{9} sin\left(2k\frac{\pi}{6k}\right) = \frac{\pi\sqrt{3}}{9} sin\left(\frac{\pi}{3}\right) = \frac{\pi\sqrt{3}}{9} \cdot \frac{\sqrt{3}}{2} = \frac{\pi}{6}$$



Relativno kretanje tačka M ima brzinu i ubrzanja

$$V_R = \dot{s}_R = \frac{a\pi\sqrt{3}}{9} 2kcos(2kt)$$

$$a_T = \ddot{s}_R = -\frac{a\pi\sqrt{3}}{9} 2k2ksin(2kt) = -\frac{a\pi\sqrt{3}}{9} 4k^2 sin(2kt)$$

$$a_N = \frac{\dot{s}_R{}^2}{a} = \frac{\left(\frac{a\pi\sqrt{3}}{9} 2kcos\,(2kt)\right)^2}{a} = \frac{a^2\pi^2 3}{81\,a} 4k^2 cos^2(2kt)$$

U trenutku $T = \frac{\pi}{6k}$

$$V_{R1} = \frac{a\pi\sqrt{3}}{9} 2kcos(2kt) = \frac{a\pi\sqrt{3}}{9} 2kcos\left(2k\frac{\pi}{6k}\right) = \frac{a\pi\sqrt{3}}{9} 2kcos\left(\frac{\pi}{3}\right) = a\pi k\frac{\sqrt{3}}{9}$$

$$a_{T1} = -\frac{a\pi\sqrt{3}}{9} 4k^2 sin(2kt) = -\frac{a\pi\sqrt{3}}{9} 4k^2 sin\left(2k\frac{\pi}{6k}\right) = -\frac{a\pi\sqrt{3}}{9} 4k^2 sin\left(\frac{\pi}{3}\right)$$

$$a_{T1} = -\frac{2ak^2\pi}{3}$$

$$\vec{a}_{T1} = a_T cos\frac{\pi}{3}\,\vec{j} + a_T sin\frac{\pi}{3}\,\vec{k} = \frac{ak^2\pi}{3}\cdot\frac{1}{2}\,\vec{j} + \frac{ak^2\pi}{3}\cdot\frac{\sqrt{3}}{2}\,\vec{k}$$

$$\vec{a}_{T1} = \frac{ak^2\pi}{3}\,\vec{j} + \frac{ak^2\pi\sqrt{3}}{3}\,\vec{k}$$

$$a_{N1} = \frac{a^2\pi^2 3}{81\,a} 4k^2 cos^2(2kt) = \frac{a\pi^2 3}{81} 4k^2 cos^2\left(2k\frac{\pi}{6k}\right) = \frac{a\pi^2 3}{81\,a} 4k^2 cos^2\left(\frac{\pi}{3}\right)$$

$$a_{N1} = a\pi^2 k^2\frac{3}{81}$$

$$\vec{a}_{N1} = -a_N cos\frac{\pi}{6}\,\vec{j} + a_N sin\frac{\pi}{6}\,\vec{k} = -a\pi^2 k^2\frac{3}{81}\frac{\sqrt{3}}{2}\,\vec{j} + a\pi^2 k^2\frac{3}{81}\frac{1}{2}\,\vec{k}$$

$$\vec{a}_{N1} = -a\pi^2 k^2\frac{\sqrt{3}}{54}\,\vec{j} + a\pi^2 k^2\frac{1}{54}\,\vec{k}$$

$$\vec{a}_R = \vec{a}_{T1} + \vec{a}_{N1} = \left(\frac{ak^2\pi}{3} - a\pi^2 k^2\frac{\sqrt{3}}{54}\right)\vec{j} + \left(\frac{ak^2\pi\sqrt{3}}{3} + a\pi^2 k^2\frac{1}{54}\right)\vec{k}$$

$$\vec{a}_R = \left(\frac{18-\pi\sqrt{3}}{54}\right) ak^2\pi\vec{j} + \left(\frac{18\sqrt{3}+\pi}{54}\right) a\pi k^2\vec{k}$$

$$\vec{V}_{R1} = V_{R1} sin\frac{\pi}{6}\,\vec{j} + V_{R1} cos\frac{\pi}{6}\,\vec{k} = a\pi k\frac{\sqrt{3}}{9}\frac{1}{2}\,\vec{j} + a\pi k\frac{\sqrt{3}}{9}\frac{\sqrt{3}}{2}\,\vec{k}$$

$$\vec{V}_{R1} = a\pi k\frac{\sqrt{3}}{18}\,\vec{j} + a\pi k\frac{1}{6}\,\vec{k}$$

Prenosno kretanje tačke M je rotacija oko z ose i brzine i ubrzanja su

$$V_P = (2a - a\,cos\varphi)\cdot\omega_0$$

$$\vec{a}_P = \vec{a}_{P\varepsilon} + \vec{a}_{P\omega}$$

$$a_{P\varepsilon} = a\frac{d\omega_0}{dt} = 0$$

$$a_{P\omega} = (2a - a\,cos\varphi)\cdot{\omega_0}^2$$

U trenutku $T = \frac{\pi}{6k}$ $\qquad \rightarrow \varphi_1 = \frac{\pi}{6}$

$$\vec{\omega}_P = -\omega_0\vec{k}$$

$$\vec{V}_P = -(2a - a\,cos\varphi_1)\cdot\omega_0\,\vec{\imath} = -\left(2-\frac{\sqrt{3}}{2}\right)\,a\,\omega_0\,\vec{\imath}$$

$$\vec{a}_P = (2a - a\,cos\varphi_1)\cdot\omega^2{}_0\,\vec{\imath} = \left(2-\frac{\sqrt{3}}{2}\right)\,a\,\omega^2{}_0\,\vec{\jmath}$$

$$\vec{a}_C = 2\vec{\omega}_P\times\vec{V}_R = \begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 0 & 0 & -2\omega_0\\ 0 & a\pi k\frac{\sqrt{3}}{18} & a\pi k\frac{1}{6}\end{vmatrix} = 2\omega_0\cdot a\pi k\frac{\sqrt{3}}{18}\,\vec{\imath} = a\pi k\frac{\sqrt{3}}{9}\,\omega_0\vec{\imath}$$

$$\vec{a}_C = a\pi k\frac{\sqrt{3}}{9}\,\omega_0\vec{\imath}$$

Apsolutna brzina tačke M u trenutku $T = \frac{\pi}{6k}$

$$\vec{V} = \vec{V}_P + \vec{V}_R$$

$$\vec{V}_P = -\left(2-\frac{\sqrt{3}}{2}\right)\,a\,\omega_0\,\vec{\imath}$$

$$\vec{V}_{R1} = a\pi k\frac{\sqrt{3}}{18}\,\vec{\jmath} + a\pi k\frac{1}{6}\,\vec{k}$$

$$\vec{V} = -\left(2-\frac{\sqrt{3}}{2}\right)\,a\,\omega_0\,\vec{\imath} + a\pi k\frac{\sqrt{3}}{18}\,\vec{\jmath} + a\pi k\frac{1}{6}\,\vec{k}$$

Apsolutno ubrzanje tačke M u trenutku $T = \frac{\pi}{6k}$

$$\vec{a} = \vec{a}_P + \vec{a}_R + \vec{a}_C$$

$$\vec{a}_P = \left(2-\frac{\sqrt{3}}{2}\right)\,a\,\omega^2{}_0\,\vec{\jmath}$$

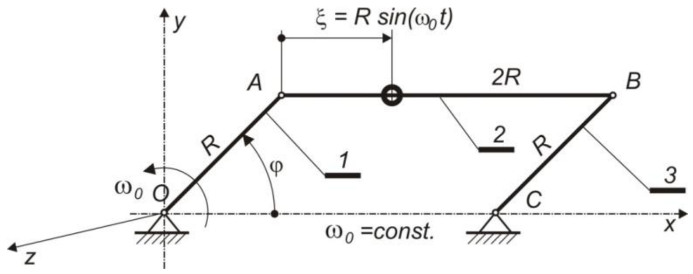
$$\vec{a}_R = \left(\frac{18-\pi\sqrt{3}}{54}\right)ak^2\pi\vec{\jmath} + \left(\frac{18\sqrt{3}+\pi}{54}\right)a\pi k^2\vec{k}$$

$$\vec{a}_C = a\pi k\frac{\sqrt{3}}{9}\,\omega_0\vec{\imath}$$

$$\vec{a} = a\pi k\frac{\sqrt{3}}{9}\,\omega_0\vec{\imath} + \left[\left(\frac{18-\pi\sqrt{3}}{54}\right)ak^2\pi + \left(2-\frac{\sqrt{3}}{2}\right)a\omega_0^2\right]\vec{\jmath} + \left(\frac{18\sqrt{3}+\pi}{54}\right)a\pi k^2\vec{k}$$

Zadatak 6. ISPITNI

Prikazani ravan mehanizam sastoji se od poluge OA(označne sa 1),dužine R, zatim poluge AB (označene sa 2) dužine 2R i poluge BC (označene sa 3) dužine R. U tačkama O,A,B,C su zglobne veze. Poluga OA se okreće konstantnom ugaonom brzinom $\omega_0 = const$ oko nepokretne ose upravne na ravan mehanizma. Zglobovi O i C su na pravoj koja je paralelna sa štapom AB.

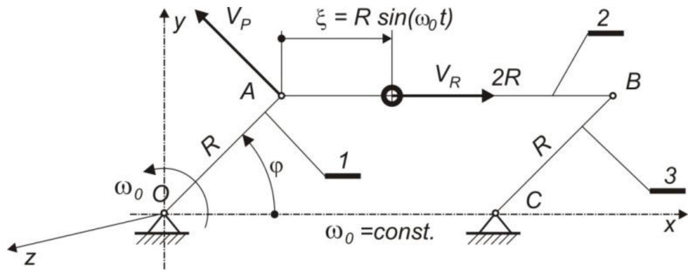


Duž poluge AB (označene sa 2) kreće se alka-tačka M po zakonu $\xi = Rsin\omega_0 t$. U početnom trenutku alka M se nalazi u tački A, a poluge OA i BC su horizontalne, odnosno $\varphi_0 = 0$

Odrediti apsolutnu brzinu i ubrzanje tačke M u trenutku kada je tačka M na sredini štapa AB.

Rešenje:

Kretanje tačke M duž poluge je relativno kretanje. Kretanje je pravolinijsko duž poluge AB. Kako su poluge OA i BC jednake i pravci OA i AB su paralelni poluga izvodi čisto translatorno kretanje, pa su relativna brzina i relativno ubrzanje u pravcu x ose.



$$\vec{V}_R = \dot{\xi}\vec{\imath} = \frac{d}{dt}\left(Rsin(\omega_0 t)\right)\vec{\imath} = R\omega_0 cos(\omega_0 t)\vec{\imath}$$

$$\vec{a}_R = \ddot{\xi}\vec{\imath} = \frac{d}{dt}\left(R\omega_0 cos(\omega_0 t)\right)\vec{\imath} = -R\omega^2{}_0 sin(\omega_0 t)\vec{\imath}$$

Prenosno kretanje je translatorno kretanje poluge AB pa su brzine i ubrzanja svih tačaka poluge AB (poluge 2) jednake

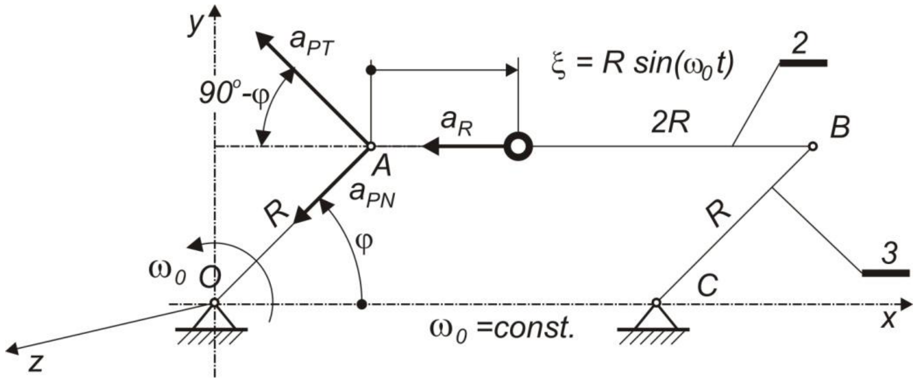
$$\varphi = \int \omega_0 \, dt = \omega_0 t + C$$

$$C = 0 \ \ jer \ je \ \varphi_0 = 0$$

$$\varphi = \omega_0 t$$

$$\omega_0 = const. \rightarrow \ \varepsilon_0 = \frac{d\omega_0}{dt} = 0$$

$$\vec{V}_P = -R\omega_0 sin\varphi\,\vec{i} + R\omega_0 cos\varphi\,\vec{j} = R\omega_0 sin(\omega_0 t)\,\vec{i} + R\omega_0 cos(\omega_0 t)\,\vec{j}$$



Apsolutna brzina tačke M

$$\vec{V} = \vec{V}_P + \vec{V}_R$$

$$\vec{V}_P = -R\omega_0 sin(\omega_0 t)\,\vec{i} + R\omega_0 cos(\omega_0 t)\,\vec{j}$$

$$\vec{V}_R = R\omega_0 cos(\omega_0 t)\vec{i}$$

$$\vec{V} = [-R\omega_0 sin(\omega_0 t) + R\omega_0 cos(\omega_0 t)]\vec{i} + R\omega_0 cos(\omega_0 t)\,\vec{j}$$

Prenosno ubrzanje

$$\vec{a}_P = \vec{a}_{PT} + \vec{a}_{PN}$$

$$a_{PT} = R\frac{d\omega_0}{dt} = 0$$

$$a_{PN} = R\omega_0^2$$

$$\vec{a}_P = -R\omega_0^2 cos\varphi\,\vec{i} - R\omega_0^2 sin\varphi\,\vec{j} = -R\omega_0^2 cos(\omega_0 t)\,\vec{i} - R\omega_0^2 sin(\omega_0 t)\,\vec{j}$$

Koriolisovo ubrzanje

$$\vec{a}_C = 0$$

Apsolutno ubrzanje

$$\vec{a} = \vec{a}_P + \vec{a}_R + \vec{a}_C$$

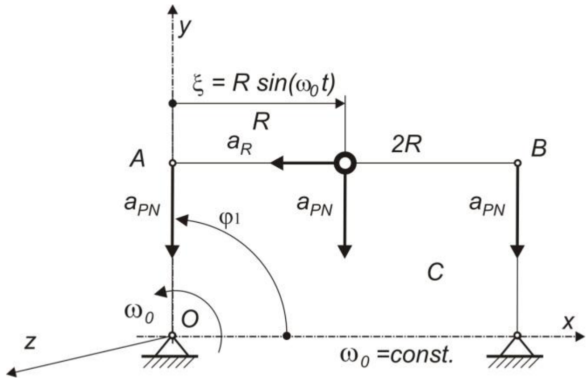
$$\vec{a}_P = -R\omega_0^2 cos(\omega_0 t)\,\vec{i} - R\omega_0^2 sin(\omega_0 t)\,\vec{j}$$

$$\vec{a}_R = -R\omega^2{}_0 sin(\omega_0 t)\vec{i}$$

$$\vec{a}_C = 0$$

$$\vec{a} = [-R\omega^2{}_0 sin(\omega_0 t) - R\omega_0^2 cos(\omega_0 t)]\,\vec{i} + [\,-R\omega_0^2 sin(\omega_0 t)]\,\vec{j}$$

Kada je alka tačka M na sredini poluge AB



$\xi = R$

$\xi = Rsin(\omega_0 t) = R \rightarrow sin(\omega_0 t) = \frac{R}{R} = 1 \rightarrow \omega_0 t = \frac{\pi}{2}$

$\varphi = \omega_0 t \rightarrow \omega_0 t = \frac{\pi}{2} \rightarrow \varphi_1 = \frac{\pi}{2}$

$\vec{V}_P = -R\omega_0 sin\left(\frac{\pi}{2}\right)\vec{i} + R\omega_0 cos\left(\frac{\pi}{2}\right)\vec{j} = -R\omega_0\vec{i}$

$\vec{V}_R = R\omega_0 cos\left(\frac{\pi}{2}\right)\vec{i} = 0$

$\boxed{\vec{V} = -R\omega_0\vec{i}}$

$\vec{a}_P = -R\omega_0^2 cos\left(\frac{\pi}{2}\right)\vec{i} - R\omega_0^2 sin\left(\frac{\pi}{2}\right)\vec{j} = -R\,\omega^2{}_0\,\vec{j}$

$\vec{a}_C = 0$

$\vec{a}_R = -R\omega^2{}_0 sin\left(\frac{\pi}{2}\right)\vec{i} = -R\,\omega^2{}_0\vec{i}$

$\boxed{\vec{a} = -R\,\omega^2{}_0\vec{i} - R\,\omega^2{}_0\,\vec{j}}$