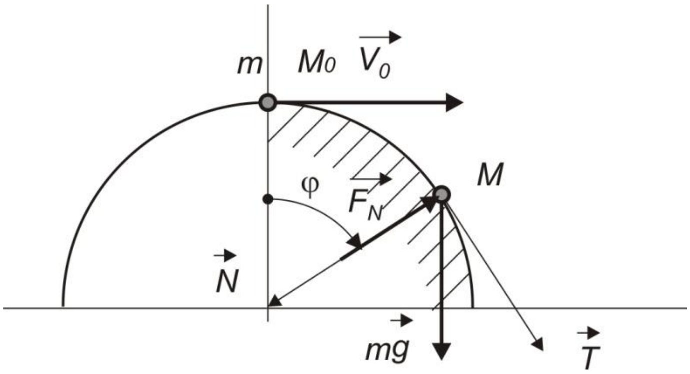
Zadatak 5.

Kamen mase m se nalazi u najvišoj tački M_0 polusferne površine poluprečnika R. Saopštena mu je početna brzina V_0 u horizontalnom pravcu. Na kom će mestu kamen napustiti vezu, kolika mora da bude početna brzina da bi kamen napustio vezu u početnom položaju? Pretpostaviti da je veza idealna.



Rešenje:

$$m\vec{a} = \sum \vec{F}_i^a + \vec{F}_W$$

Veza je idealna pa je $\vec{F}_W = \vec{F}_N$

$$m\vec{a} = m\vec{g} + \vec{F}_N$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N} = R\ddot{\varphi}\vec{T} + R\dot{\varphi}^2\vec{N}$$

$$\text{T: } ma_T = mgcos(90-\varphi) = mgsin\varphi \rightarrow \quad a_T = R\ddot{\varphi} = gsin\varphi$$

$$\text{N: } ma_N = mgcos\varphi - F_N \rightarrow \quad F_N = m(-R\dot{\varphi}^2 + gcos\varphi)$$

$$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi}\cdot\frac{d\varphi}{dt} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} \rightarrow \ddot{\varphi} = \frac{g}{R}sin\varphi \rightarrow \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} = \frac{g}{R}sin\varphi$$

$$\dot{\varphi}\, d\dot{\varphi} = \frac{g}{R}sin\varphi d\varphi \rightarrow \int \dot{\varphi}\, d\dot{\varphi} = \frac{g}{R}\int sin\varphi d\varphi$$

$$\frac{\dot{\varphi}^2}{2} = -\frac{g}{R}cos\varphi + C_1$$

$$t = 0 \rightarrow \; R\dot{\varphi}_{t=0} = V_0 \; \rightarrow \dot{\varphi}_{t=0} = \frac{V_0}{R} \rightarrow$$

$$\frac{1}{2}\left(\frac{V_0}{R}\right)^2 = -\frac{g}{R}cos0 + C_1 \rightarrow C_1 = \frac{1}{2}\left(\frac{V_0}{R}\right)^2 + \frac{g}{R} = \frac{{V_0}^2 + 2gR}{2R^2}$$

$$\dot{\varphi}^2 = -\frac{2g}{R}cos\varphi + 2\frac{{V_0}^2 + 2g}{2R}$$

$$\dot{\varphi} = \sqrt{-\frac{2g}{R}cos\varphi + 2\frac{{V_0}^2 + 2gR}{2R^2}} = \frac{1}{R}\sqrt{{V_0}^2 + 2gR(1-cos\varphi)}$$

$$V = R\dot{\varphi} = \sqrt{{V_0}^2 + 2gR(1-cos\varphi)}$$

Uslov napuštanja veze je da reakcija veze jednaka nuli

$$F_N = m(-R\dot{\varphi}^2 + gcos\varphi) = 0 \rightarrow m\left(-\frac{V^2}{R} + gcos\varphi\right) = 0$$

$$V^2 - gRcos\varphi = 0$$

$$\left[{V_0}^2 + 2gR(1 - cos\varphi)\right] - gRcos\varphi = 0$$

$${V_0}^2 + 2gR(1 - cos\varphi) - gRcos\varphi = 0$$

$$2gR - 2gRcos\varphi - gRcos\varphi = -{V_0}^2$$

$$2 - 3cos\varphi = \frac{-{V_0}^2}{gR} \rightarrow cos\varphi = \frac{2}{3} + \frac{{V_0}^2}{3gR} \rightarrow \varphi = arccos\left(\frac{2}{3} + \frac{{V_0}^2}{3gR}\right)$$

Uslov napuštanja veze za φ=0 je da je reakcija veze jednaka nuli, a potrebna je početna brzina

$$F_N = m(-R\dot{\varphi}^2 + gcos\varphi) = 0 \rightarrow m(-R\dot{\varphi}^2 + gcos0) = 0 \rightarrow R\dot{\varphi}^2 - g = 0$$

$$\dot{\varphi}_{t=0} = \frac{V_0}{R} \rightarrow R\frac{{V_0}^2}{R^2} - g = 0 \rightarrow {V_0}^* = \sqrt{Rg}$$

Rešenje primenom zakona održanja energije:

$m\vec{a} = \sum \vec{F}_i^a + \vec{F}_W$ *Veza je idealna pa je* $\vec{F}_W = \vec{F}_N$

$$m\vec{a} = m\vec{g} + \vec{F}_N$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N} = R\ddot{\varphi}\vec{T} + \frac{V^2}{R}\vec{N}$$

T: $ma_T = mgcos(90 - \varphi) = mgsin\varphi \rightarrow \quad a_T = R\ddot{\varphi} = gsin\varphi$

N: $ma_N = mgcos\varphi - F_N \rightarrow \quad F_N = m\left(-\frac{V^2}{R} + gcos\varphi\right)$

$$E_{K1} + E_{P1} = E_{K2} + E_{P2}$$

$$\frac{1}{2}mV_0^2 + mgR(1 - cos\varphi) = \frac{1}{2}mV^2 + 0 \rightarrow V^2$$

$$V^2 = V_0^2 + 2Rg(1 - cos\varphi)$$

$$V = \sqrt{V_0^2 + 2Rg(1 - cos\varphi)}$$

Uslov napuštanja veze je da je reakcija veze jednaka nuli

$$F_N = m(-R\dot{\varphi}^2 + gcos\varphi) = 0 \rightarrow m\left(-\frac{V^2}{R} + gcos\varphi\right) = 0$$

$$V^2 - gRcos\varphi = 0$$

$$V_0^2 + 2Rg(1 - cos\varphi) - gRcos\varphi = 0$$

$$2gR - 2gRcos\varphi - \ gRcos\varphi = -{V_0}^2$$

$$2 - 3cos\varphi = \frac{-{V_0}^2}{gR} \rightarrow cos\varphi = \frac{2}{3} + \frac{{V_0}^2}{3gR} \rightarrow \varphi = arccos\left(\frac{2}{3} + \frac{{V_0}^2}{3gR}\right)$$

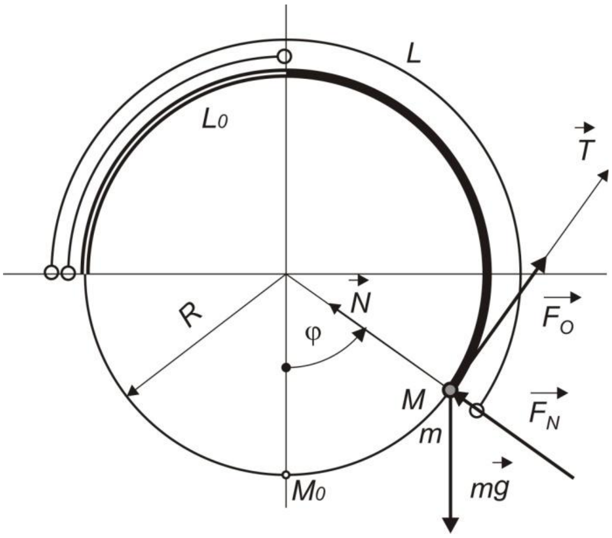
Uslov napuštanja veze za φ=0 je da je reakcija veze jednaka nuli, a potrebna je početna brzina

$$F_N = m\left(-\frac{V^2}{R} + gcos\varphi\right) = 0 \rightarrow m\left(-\frac{V^2}{R} + gcos0\right) = 0 \rightarrow \frac{V^2}{R} - g = 0$$

$$\rightarrow {V_0}^* = \sqrt{Rg}$$

Zadatak 6.

Po glatkoj vezi kružnog oblika poluprečnika R kreće se u vertikalnoj ravni prsten M mase m zanemarljivih dimenzija. Opruga krutosti c vezana je jednim krajem za nepomičnu tačku A, a drugim krajem za prsten M. Dužina opruge u nenapregnutom stanju je Lo=Rπ/2. Ako je prsten mirovao u položaju Mo odrediti reakciju veze zavisno od ugla φ.



Rešenje:

$$L_0 = R\frac{\pi}{2}$$

$$L = R\left(\frac{3\pi}{2} - \varphi\right)$$

$$F_o = c\Delta L = \ cR\left[\left(\frac{3\pi}{2} - \varphi\right) - \frac{\pi}{2}\right]$$

$$F_o = cR(\pi - \varphi)$$

Veza je idealna pa je $\vec{F}_W = \vec{F}_N$

$$m\vec{a} = \sum \vec{F}_i^a + \vec{F}_W$$

$$m\vec{a} = m\vec{g} + \vec{F}_N + \vec{F}_o$$

$$\vec{a} = a_T\vec{T} + a_N\vec{N} = R\ddot{\varphi}\vec{T} + \frac{V^2}{R}\vec{N}$$

Vektorska diferencijalna jednačina se projektuje na prirodni koordinatni sistem

T: $ma_T = -mgsin\varphi + F_o \rightarrow \quad a_T = R\ddot{\varphi} = -mgsin\varphi + cR(\pi - \varphi)$

N: $ma_N = -mgcos\varphi + F_N \rightarrow \quad F_N = m\left(\frac{V^2}{R} + gcos\varphi\right)$

$mR\ddot{\varphi} = -mgsin\varphi + cR(\pi - \varphi)$

$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi} \cdot \frac{d\varphi}{dt} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} \rightarrow$

$mR\dot{\varphi}\frac{d\dot{\varphi}}{d\varphi} = -mgsin\varphi + cR(\pi - \varphi)$

$\dot{\varphi}d\dot{\varphi} = \left[-\frac{g}{R}sin\varphi + \frac{c}{m}(\pi - \varphi)\right]d\varphi$

$\int \dot{\varphi}d\dot{\varphi} = \int \left[-\frac{g}{R}sin\varphi + \frac{c}{m}(\pi - \varphi)\right]d\varphi + C_1$

$\frac{1}{2}\dot{\varphi}^2 = \frac{g}{R}cos\varphi + \frac{c}{m}(\pi - \varphi)\varphi + C_1$

$t = 0 \rightarrow \; R\dot{\varphi}_{t=0} = V_0 = 0 \; \rightarrow \varphi_{t=0} = 0 \rightarrow C_1 = -\frac{g}{R}$

$\frac{1}{2}\dot{\varphi}^2 = \frac{g}{R}cos\varphi + \frac{c}{m}\left(\pi - \frac{\varphi}{2}\right)\varphi - \frac{g}{R}$

$V = R\varphi \rightarrow V^2 = R^2\varphi^2 = 2gR(cos\varphi - 1) + \frac{cR^2}{m}\left(\pi - \frac{\varphi}{2}\right)\varphi$

$$F_N = m\left(\frac{V^2}{R} + gcos\varphi\right) = \frac{m}{R}2gR(cos\varphi - 1) + \frac{m}{R}\frac{cR^2}{m}\left(\pi - \frac{\varphi}{2}\right)\varphi + mgcos\varphi$$

$F_N = 3mgcos\varphi - 2mg + cR\varphi\left(2\pi - \frac{\varphi}{2}\right)$