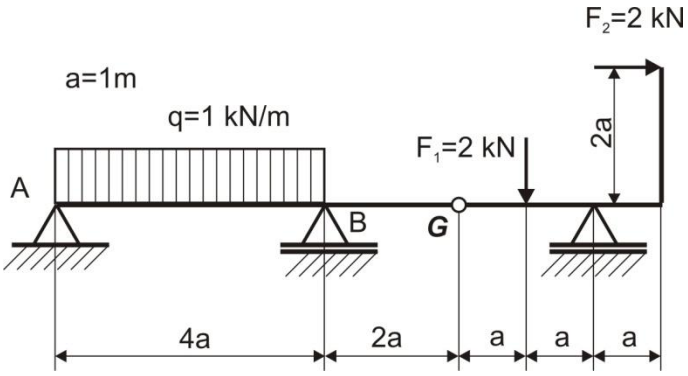
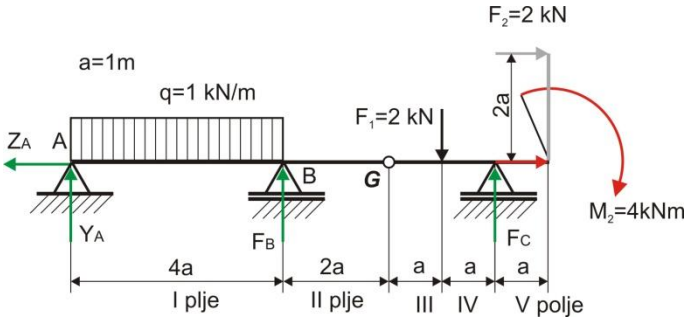
Primer 3.7

Za Gerberovu gredu prikazanu na slici odrediti otpore oslonaca i nacrtati osnovne statičke dijagrame. Poznata su opterećenja $F_1 = 2$ kN, $F_2 = 2$kN, $q = 1\ \frac{kN}{m}$, a=1m .



Da bi se rešila data greda treba paralelno premestiti silu F_2 na pravac grede što uslovljava dodavanje momenta u tački u koju se sila premešta a jednakog proizvodu normalnog rastojanja i sile. U osloncima se zamenjuje njihovo dejstvo reakcijama oslonaca, kod tačke nepokretnog oslonca A kosa sila u ravni odnosno dve komponente i u tački pokretnog oslonca B vertikalna sila.



$M_2 = F_2 \cdot 2a = 2 \cdot 2 = 4kNm$
$F_q = q \cdot 4a = 1 \cdot 4 = 4\ kN$

1. $\sum Z_i = -Z_A + F_2 = 0$
2. $\sum Y_i = Y_A - F_q - F_1 + F_B + F_C = 0$
3. $\sum \mathrm{M_A} = 2\mathrm{a} \cdot \mathrm{F_q} + 7\mathrm{a} \cdot \mathrm{F_1} + \mathrm{M_2} - \mathrm{a} \cdot \mathrm{F_2} - 4\mathrm{a} \cdot \mathrm{F_B} - 8\mathrm{a} \cdot \mathrm{F_C} = 0$
4. $\sum \mathrm{M_G^D} = -\mathrm{M_2} - \mathrm{a} \cdot \mathrm{F_1} + 2\mathrm{a} \cdot \mathrm{F_C} = 0$

$Z_A = F_2 = 2\ kN$

$F_C = \frac{\mathrm{F_1 + M_2}}{2} = \frac{2+4}{2} = 3\ kN$

$\mathrm{F_B} = \frac{1}{4a}\left(2\mathrm{a} \cdot \mathrm{F_q} + 7\mathrm{a} \cdot \mathrm{F_1} - 8\mathrm{a} \cdot \mathrm{F_{C+M_2}}\right) = \frac{1}{4}(2 \cdot 1 \cdot 4 + 7\mathrm{a} \cdot 2 - 8\mathrm{a} \cdot 3 + 4) = 0.5\ \mathrm{kN}$

$Y_A = F_q - F_B + F_1 - F_C = 4 - 0.5 + 2 - 3 = 0.5\ kN$

Provera otpora $\sum M_B = 4a \cdot Y_A - 4a \cdot F_q + 3a \cdot F_1 + M_2 + 4a \cdot F_C =$
$= 2.5 \cdot 4 - 4 \cdot 1 \cdot 2 + 3 \cdot 2 + 4 - 4 \cdot 3 = 0$

Izvršiti podelu na polja

I polje z od 0 do 4a

1. Aksijalna sila

$F_a = Z_A$ z=0 $F_a = Z_A = 2\ kN$
z=a $F_a = Z_A = 2\ kN$

2. Transverzalnalna sila

$F_T = Y_A - q \cdot z$ z=0 $F_T = Y_A = 2.5\ kN$

z=4a $F_T = Y_A - 4a \cdot q = 2.5 - 4 \cdot 1 = -1.5\ kN$

3. Moment savijanja sa leve strane

$M_f^L = Y_A \cdot z - q \cdot \frac{z^2}{2} = 2.5z - 0.5z^2$

z=0 $M_f = 0$

z=4a $M_f = Y_A \cdot 4a - q \cdot \frac{16a^2}{2} = 2.5 \cdot 4 - 1\frac{4^2}{2} = 2\ kNm$

prevojna tačka – ekstrem

$F_T = Y_A - z_0 \cdot q = 0 \rightarrow z_0 = \frac{Y_A}{q} = \frac{2.5}{1} = 2.5\ m$

$M_{fzo}^L = 2.5z_0 - 0.5{z_0}^2 = 2.5 \cdot 2.5 - 0.5 \cdot 2.5^2 = 3.125\ kNm$

II polje z od 4a do 7a

1. Aksijalna sila

$F_a = Z_A$ z=4 $F_a = Z_A = 2\ kN$

z=7 $F_a = Z_A = 2\ kN$

2. Transverzalnalna sila

$F_T = Y_A - q \cdot 4a + F_B = Y_A - F_q + F_B$

z=4a $F_T = Y_A - F_q + F_B = 2.5 - 4 + 0.5 = -1\ kN$

z=7a $F_T = Y_A - F_q + F_B = 2.5 - 4 + 0.5 = -1\ kN$

3. Moment savijanja sa leve strane

$M_f^L = Y_A \cdot z - F_q(z - 2a) + F_B(z - 4a) = 2.5z - 4z + 8 + 0.5z - 2 = -z + 6$

z=4a $M_f = -z + 6 = -4 + 6 = 2\ kNm$

z=7a $M_f = -z + 6 = -7 + 6 = -1\ kNm$

III polje z od 7a do 8a

1. Aksijalna sila

$F_a = Z_A$ z=4 $F_a = Z_A = 2\ kN$

z=7 $F_a = Z_A = 2\ kN$

2. Transverzalnalna sila

$F_T = Y_A - F_q + F_B - F_1 = 2.5 - 4 + 0.5 - 2 = -3\ kN$

z=7a $F_T = -3\ kN$

z=8a $F_T = -3\ kN$

3. Moment savijanja sa desne strane

$M_f^D = M_f = F_C(8a - z) - M_2 = 24 - 3z - 4 = 20 - 3z$

z=7a $M_f^D = 22 - 3z = 20 - 3 \cdot 7 = -1\ kNm$

z=8a $M_f^D = 22 - 3z = 20 - 3 \cdot 8 = -4\ kNm$

IV polje z od 8a do 9a

1. Aksijalna sila

$F_a = Z_A$ z=8 $F_a = Z_A = 2\ kN$

z=9 $F_a = Z_A = 2\ kN$

2. Transverzalnalna sila

$F_T = Y_A - F_q + F_B - F_1 + F_C = 2.5 - 4 + 0.5 - 2 + 3 = 0$

z=8a $F_T = 0$

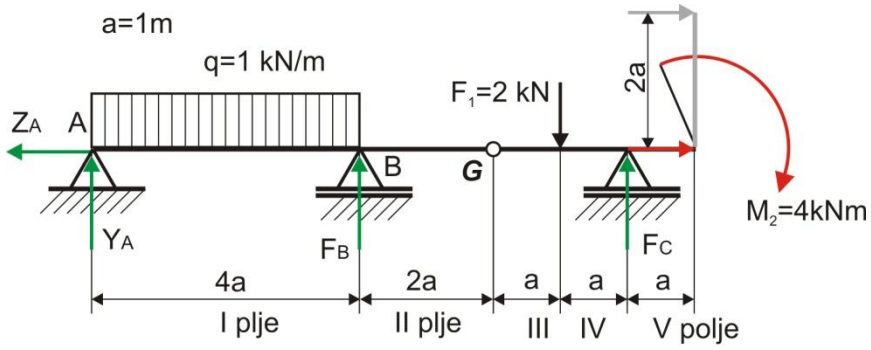
z=9a $F_T = 0$

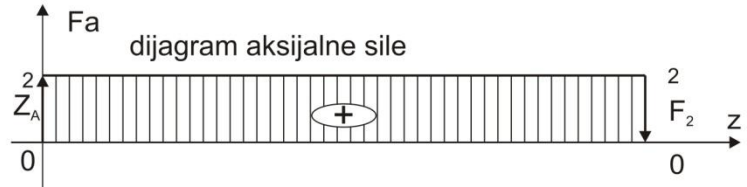
3. Moment savijanja sa leve strane

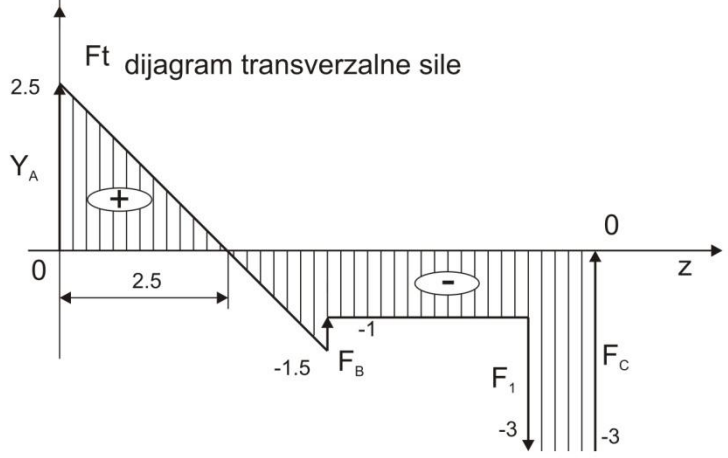
$M_f^D = -M_2 = -4\ kNm$

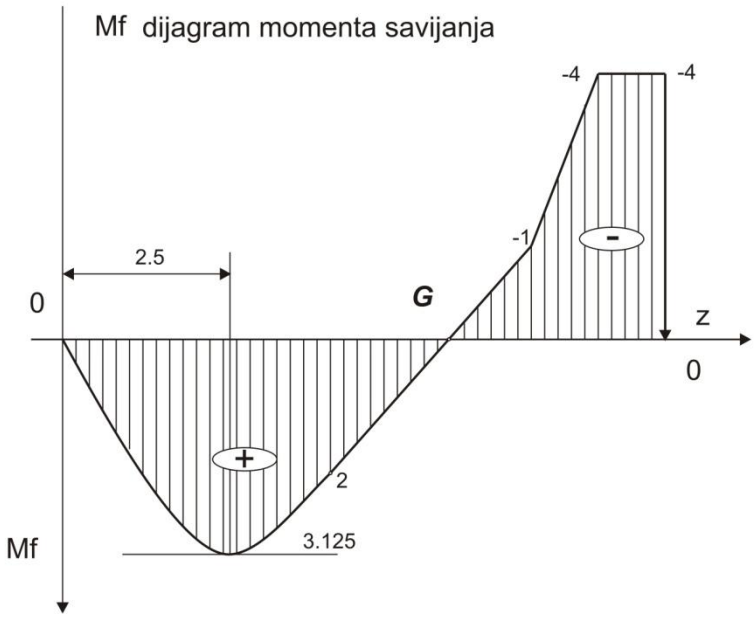
z=8a $M_f^D = -M_2 = -4\ kNm$

z=9a $M_f^D = -M_2 = -4\ kNm$



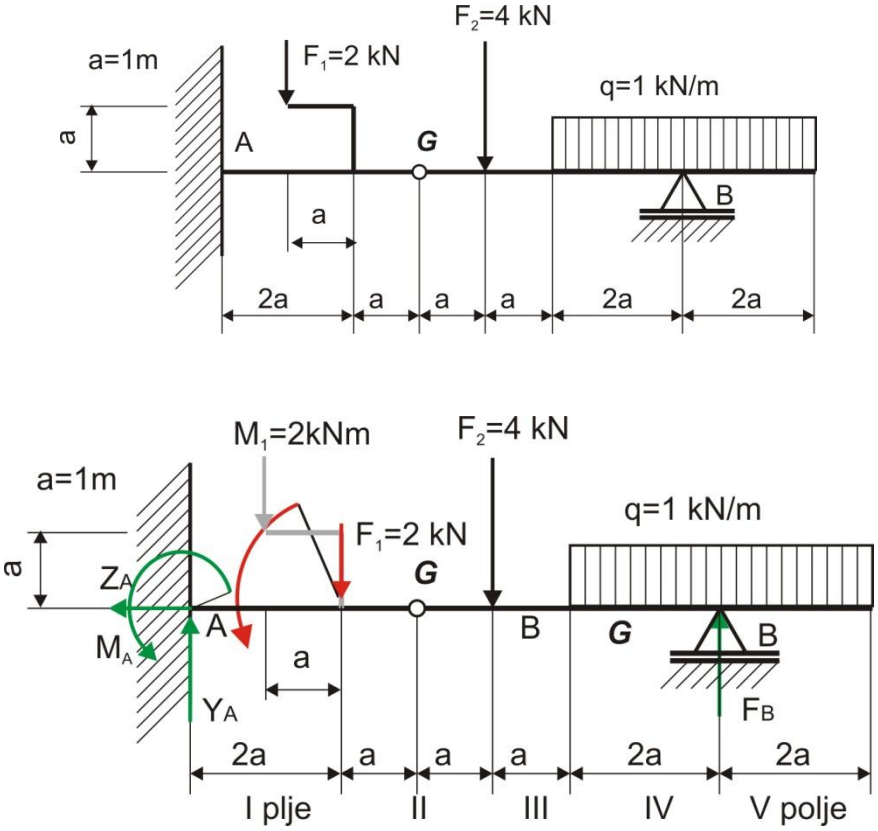






Primer 3.8

Za konzolu sa Gerberovim zglobom prikazanu na slici odrediti otpore oslonaca i nacrtati osnovne statičke dijagrame. Poznata su opterećenja $F_1 = 2$ kN, $F_2 = 4$kN, $q = 1\ \frac{kN}{m}$, a=1m .



$M_1 = F_1 \cdot a = 2 \cdot 1 = 2\ kNm$

$F_q = q \cdot 4a = 1 \cdot 4 = 4\ kN$

1. $\sum Z_i = -Z_A = 0$
2. $\sum Y_i = Y_A - F_q - F_1 - F_2 + F_B = 0$
3. $\sum M_A = -M_A - M_1 + 2a \cdot F_1 + 4a \cdot F_2 + 7a \cdot F_q - 7a \cdot F_B = 0$
4. $\sum M_G^D = 1a \cdot F_2 + 4a \cdot F_q - 4a \cdot F_B = 0$

$Z_A = 0$

$F_B = \frac{1}{4a}\left(1a \cdot F_2 + 4a \cdot F_q\right) = \frac{1}{4 \cdot 1}(1 \cdot 4 + 4 \cdot 1 \cdot 4) = 5\ kN$

$Y_A = F_1 + F_2 + F_q - F_B = 2 + 4 + 4 - 5 = 5\ kN$

$M_A = -M_1 + F_1 \cdot 2a + 4a \cdot F_2 + 7a \cdot F_q - 7a \cdot F_B =$

$M_A = -2 + 2 \cdot 2 \cdot 1 + 4 \cdot 1 \cdot 4 + 7 \cdot 1 \cdot 4 - 7 \cdot 1 \cdot 5 = 11\ kNm$

Provera otpora oslonaca

$\sum M_B = M_A - Y_A \cdot 7a + M_1 + 5a \cdot F_1 + 3a \cdot F_2 = 11 - 35 + 2 + 10 + 12 = 0$

Podela konzole sa Gerberovim zglobom na polja

Polje I $0 \leq z \leq 2a$

1. Promena aksijalne sile i dijagram aksijalne sile
   $F_a = 0$
2. Promena transverzalne sile(sa leve strane) i dijagram transverzalne sile

$F_T = Y_A = 5\ kN$

3. Promena momenta savijanja(sa leve strane) i dijagram momenta savijanja

$M_f = -M_A + Y_A \cdot z = -11 + 5 \cdot z$

vrednost su za z=0: Mf(z=0) =0
vrednost su za z=2a=2: Mf(z=2) =-1 kNm

Polje II $2a \leq z \leq 4a$

1. Promena aksijalne sile i dijagram aksijalne sile
   $F_a = 0$
2. Promena transverzalne sile(sa leve strane) i dijagram transverzalne sile

$F_T = Y_A - F_1 = 5 - 2 = 3\ kN$

vrednost su za z=2a=2: F_T(z=1) =3 kN
vrednost su za z=4a=4: F_T(z=2) =3 kN

3. Promena momenta savijanja(sa leve strane) i dijagram momenta savijanja

$M_f = -M_A + Y_A \cdot z - M_1 - F_1(z - 2a) = -11 + 5z - 2 - 2z + 4 = -9 + 3z$

vrednost su za z=2a=1: Mf(z=2) =-3 kNm
vrednost su za z=4a=4: Mf(z=4) =3 kNm

Polje III $4a \leq z \leq 5a$

1. Promena aksijalne sile i dijagram aksijalne sile
   $F_a = 0$
2. Promena transverzalne sile(sa leve strane) i dijagram transverzalne sile

$F_T = Y_A - F_1 - F_2 = 5 - 2 - 4 = -1\ kN$

vrednost su za z=4a=4: F_T(z=2) =-1 kN
vrednost su za z=5a=5: F_T(z=3) =-1 kN

3. Promena momenta savijanja(sa leve strane) i dijagram momenta savijanja

$M_f = -M_A + Y_A \cdot z - M_1 - F_1(z - 2a) - F_2(z - 4a) =$

$M_f^L - 11 + 5z - 2 - 2z + 4 - 4z + 16 = 7 - z$

vrednost su za z=4a=4: Mf(z=4) =3 kNm
vrednost su za z=5a=5: Mf(z=5) =2kNm

Polje IV $5a \leq z \leq 7a$

1. Promena aksijalne sile i dijagram aksijalne sile

$F_a = 0$

2. Promena transverzalne sile(sa desne strane) i dijagram transverzalne sile

$F_T^D = -F_B + q(9a - z) = -5 - z + 9 = 4 - z$

vrednost su za z=5a=5: F_T(z=5) = -1 kN

vrednost su za z=7a=7: F_T(z=7) = -3 kN

3. Promena momenta savijanja(sa desne strane) i dijagram momenta savijanja

$M_f^d = F_B \cdot (7a - z) - \frac{1}{2}q(9a - z)^2 = 35 - 5z - 40.5 + 9z - 0.5z^2 = -5.5 + 4z - 0.5z^2$

vrednost su za z=5a=5: Mf(z=5) =2 kNm

vrednost su za z=4a=4: Mf(z=4) =-2 kNm

Polje V $4 \leq z \leq 5a$

1. Promena aksijalne sile i dijagram aksijalne sile

$F_a = Z_A - Z_1 = 0$

2. Promena transverzalne sile(sa desne strane) i dijagram transverzalne sile

$F_T^D = q(9a - z) = 9 - z\ kN$

vrednost su za z=7a=7: F_T(z=7) = 2 kN

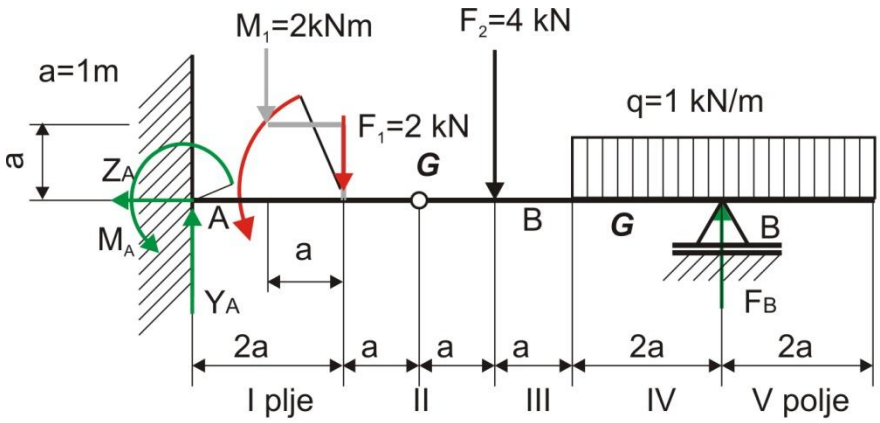
vrednost su za z=9a=9: F_T(z=9) = 0 kN

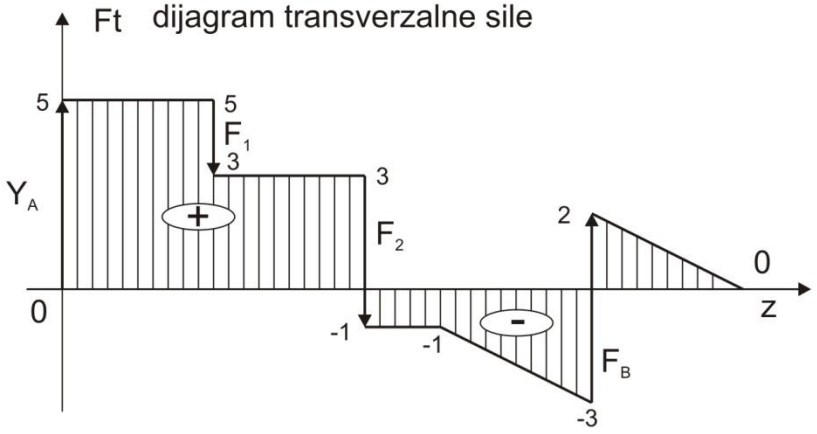
3. Promena momenta savijanja(sa desne strane) i dijagram momenta savijanja

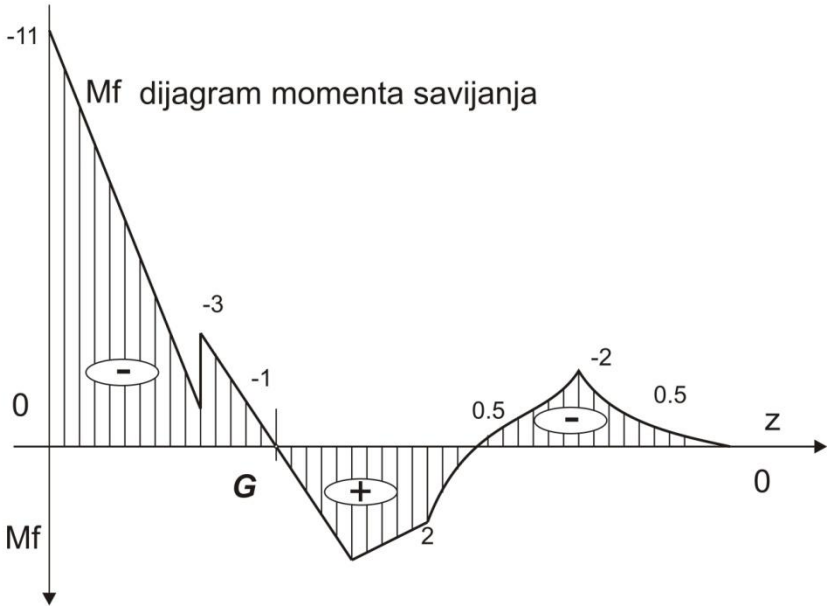
$M_f^d = -\frac{1}{2}q(9a - z)^2 = -40.5 + 9z - 0.5z^2$

vrednost su za z=7a=7: Mf(z=7) = 2 kNm

vrednost su za z=9a=9: Mf(z=9) =0


M1=2kNm
F2=4 kN
a=1m
q=1 kN/m
F1=2 kN
a
ZA
G
A
B
G
B
MA
a
YA
FB
2a
a
a
a
2a
2a
I plje
II
III
IV
V polje


Ft dijagram transverzalne sile
5
5
F1
3
YA
3
F2
2
0
0
z
-1
-1
FB
-3


-11
Mf dijagram momenta savijanja
-3
-1
-2
0.5
0.5
0
z
G
0
Mf
2