PROIZVOLJAN PROSTORNI SISTEM SILA

Jedan od zadataka statike je svođenje sistema sila i spregova na jednostavniji oblik određivanjem glavnog vektora sila (rezultantu sistema sila) i određivanjem glavnog momenta (rezultujućeg momenta sila i spregova).

Kod proizvoljnog ravnog sistema sila i momenata

$F_R = \sum_{i=1}^{n} F_i$ i $\mathfrak{Mo} = \sum_{i=1}^{n} M_i$

$$X_R = \sum_{i=1}^{n} X_i \quad \text{i} \quad Y_R = \sum_{i=1}^{n} F_i$$

Uslovi ravnoteže proizvoljnog ravnog sistema sila da su glavni vektor i glavni moment jednaki nuli

$$\sum_{i=1}^{n} X_i = 0$$

$$\sum_{i=1}^{n} Y_i = 0$$

$$\sum_{i=1}^{n} M_O^{\vec{F}} = 0$$

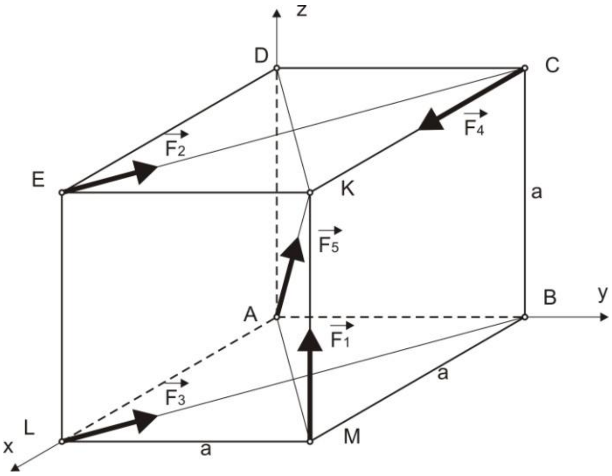
Teorema o paralelnom prenošenju sila

Dejstvo sile F na kruto telo se ne menja ako je prenesemo paralelno samoj sebi u bilo koju drugu tačku tela i pri tom pridodamo spreg sila čiji je moment jednak momentu sile koju paralelno prenosimo u odnosu na tačku u koju se sila prenosi.

Teorema se može dokazati primenom **drugog aksioma** da se telu u ravnoteži može pridodati ili oduzeti uravnoteženi sistem sila. Dodaje se paralelna sila istog intenziteta i smera u tački u koju želimo da prenesemo silu i oduzima u toj tački ista sila suprotnog smera.

Primer 3.1

Na pravougli paralelopiped strana a=b=c=10cm, dejstvuju sile čiji intenziteti su F_1= F_2= F_3= F_4= F_5=10 daN. Redukovati ovaj siatem u tačku A.



$$\overrightarrow{F_1} = (0,0,F_1) = (0,0,10)$$

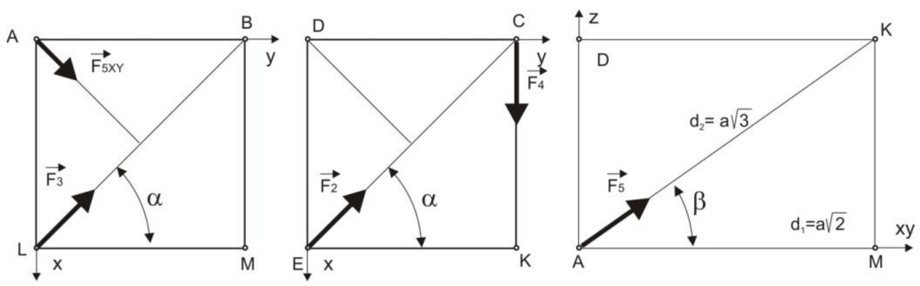
$$\overrightarrow{F_2} = (-F_2 \sin\alpha\,, F_2 \cos\alpha\,, 0)$$

$$\overrightarrow{F_2} = \left(-\frac{F_2\sqrt{2}}{2}, \frac{F_2\sqrt{2}}{2}, 0\right) = \left(-\frac{10\sqrt{2}}{2}, \frac{10\sqrt{2}}{2}, 0\right)$$

$$\overrightarrow{F_3} = (-F_3 \sin\alpha\,, F_3 \cos\alpha\,, 0)$$

$$\overrightarrow{F_3} = \left(-\frac{F_3\sqrt{2}}{2}, \frac{F_{23}\sqrt{2}}{2}, 0\right) = \left(-\frac{10\sqrt{2}}{2}, \frac{10\sqrt{2}}{2}, 0\right)$$

$$\overrightarrow{F_4} = (F_4, 0,0) = (10,0,0)$$



$$\overrightarrow{F_5} = (F_5 \cos\beta \cos\alpha\,, F_5 \cos\beta \sin\alpha\,, F_5 \cos\beta\ )$$

$$\overrightarrow{F_5} = \left(F_5 \frac{\sqrt{3}}{3}, F_5 \frac{\sqrt{3}}{3}, F_5 \frac{\sqrt{3}}{3}\right) = \left(10 \frac{\sqrt{3}}{3}, 10 \frac{\sqrt{3}}{3}, 10 \frac{\sqrt{3}}{3}\right)$$

$$\sin\alpha = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\alpha = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin\beta = \frac{a}{a\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cos\beta = \frac{a\sqrt{2}}{a\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\overrightarrow{r_1} = \overrightarrow{r_M} = (a, a, 0)$$

$$\overrightarrow{r_2} = \overrightarrow{r_E} = (a, 0, a)$$

$$\overrightarrow{r_3} = (a, 0,0)$$

$$\overrightarrow{r_4} = \overrightarrow{r_C} = (0, a, a)$$

$$\overrightarrow{r_5} = \overrightarrow{r_A} = (0,0,0)$$

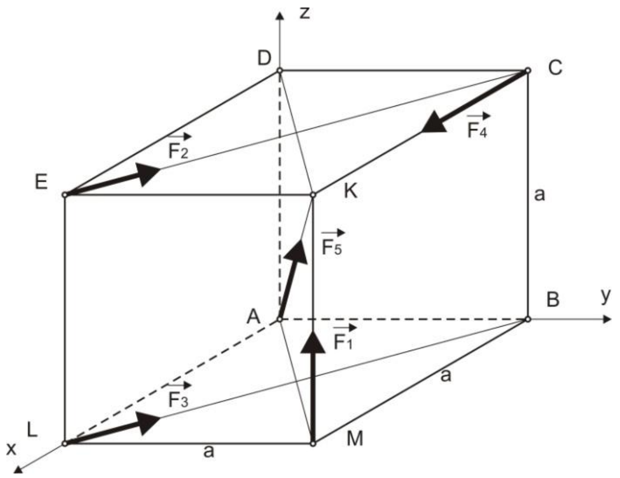
$X_R = \sum X_i = -F_2 \frac{\sqrt{2}}{2} - F_3 \frac{\sqrt{2}}{2} + F_4 + F_5 \frac{\sqrt{3}}{3} = 1.63\ daN$

$Y_R = \sum Y_i = F_2 \frac{\sqrt{2}}{2} + F_{32} \frac{\sqrt{2}}{2} + F_5 \frac{\sqrt{3}}{3} = 19.92\ daN$

$Z_R = \sum Z_i = F_1 + F_5 \frac{\sqrt{3}}{3} = 15.77\ daN$

$\overrightarrow{F_R} = X_R \vec{\imath} + Y_R \vec{\jmath} + Z_R \vec{k} = 1.63\vec{\imath} + 19.92\vec{\jmath} + 15.77\vec{k}$

$$\left|\overrightarrow{F_R}\right| = F_R = \sqrt{1.63^2 + 19.92^2 + 15.77^2} = 25.46\ daN$$



Momenti za ose

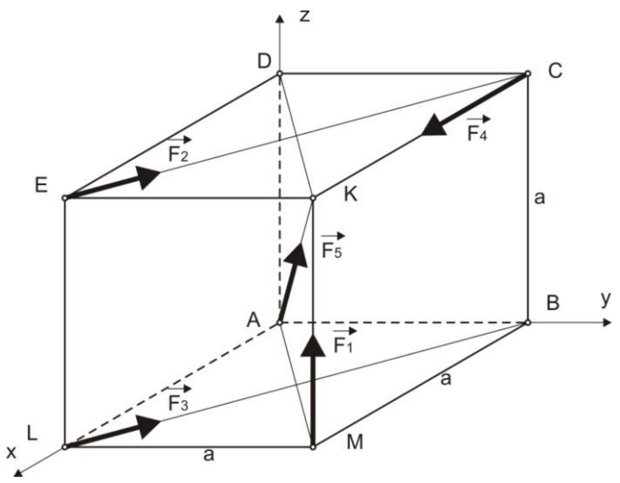
$\sum M_x = F_1\, a - F_2 a \cos\alpha = 100\left(1 - \frac{\sqrt{2}}{2}\right)$ =29.3 daNcm

$\sum M_y = -F_1\, a - F_2 a \sin\alpha + F_4 a = 100\left(1 - 1 - \frac{\sqrt{2}}{2}\right)$ =70.7 daNcm

$\sum M_z = F_2 a \sin\alpha + F_3 a \cos\alpha - F_4 a = 100\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right) = 100\left(\sqrt{2} - 1\right)$ =41.4 daNcm

$$\overrightarrow{M_A} = M_x \vec{\imath} + M_y \vec{\jmath} + M_z \vec{k} = 29.3\vec{\imath} + 70.7\vec{\jmath} + 41.4\vec{k}$$

$\left|\overrightarrow{M_A}\right| = M_A = \sqrt{29.3^2 + 70.7^2 + 41.4^2} = 87\ daNcm$



Drugi način određivanja momenata za ose

$$\overrightarrow{M_A^{F_1}} = \vec{r_1} \times \vec{F_1} = \vec{r_M} \times \vec{F_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & a & 0 \\ 0 & 0 & F_1 \end{vmatrix} = F_1 a\vec{i} - F_1 a\vec{j}$$

$$\overrightarrow{M_A^{F_2}} = \vec{r_2} \times \vec{F_2} = \vec{r_E} \times \vec{F_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 0 & a \\ -F_2\frac{\sqrt{2}}{2} & F_2\frac{\sqrt{2}}{2} & 0 \end{vmatrix} = -F_2\frac{\sqrt{2}}{2}a\vec{i} - F_2\frac{\sqrt{2}}{2}a\vec{j} + F_2\frac{\sqrt{2}}{2}a\vec{k}$$

$$\overrightarrow{M_A^{F_3}} = \vec{r_3} \times \vec{F_3} = \vec{r_L} \times \vec{F_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 0 & 0 \\ -F_3\frac{\sqrt{2}}{2} & F_3\frac{\sqrt{2}}{2} & 0 \end{vmatrix} = F_3\frac{\sqrt{2}}{2}a\vec{k}$$

$$\overrightarrow{M_A^{F_4}} = \vec{r_4} \times \vec{F_4} = \vec{r_C} \times \vec{F_4} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & a \\ F_4 & 0 & 0 \end{vmatrix} = F_4 a\vec{j} - F_4\vec{k}$$

$$\overrightarrow{M_A^{F_5}} = \vec{r_5} \times \vec{F_3} = \vec{r_A} \times \vec{F_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 0 \\ F_3\frac{\sqrt{2}}{2} & F_3\frac{\sqrt{2}}{2} & F_3\frac{\sqrt{2}}{2} \end{vmatrix} = 0$$

$$\overrightarrow{M_A} = M_x\vec{i} + M_y\vec{j} + M_z\vec{k} = 29.3\vec{i} + 70.7\vec{j} + 41.4\vec{k}$$

$$\left|\overrightarrow{M_A}\right| = M_A = \sqrt{29.3^2 + 70.7^2 + 41.4^2} = 87\ daNcm$$