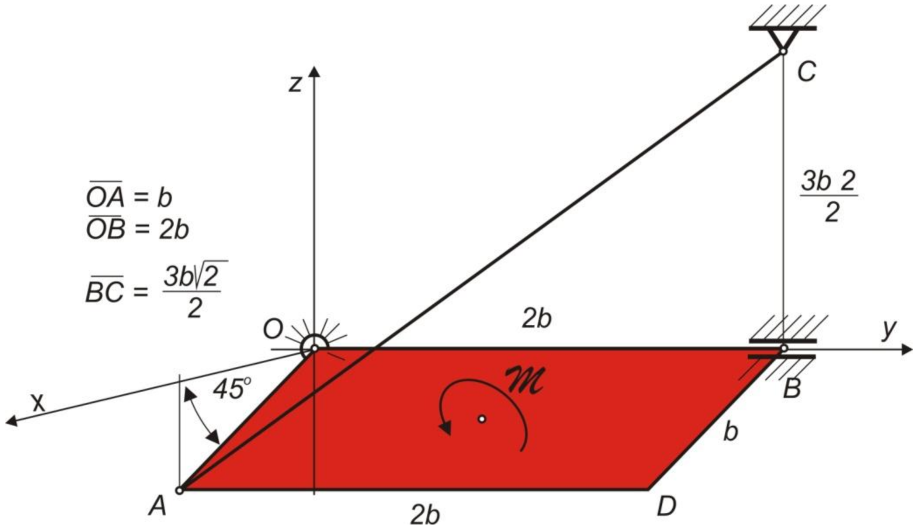
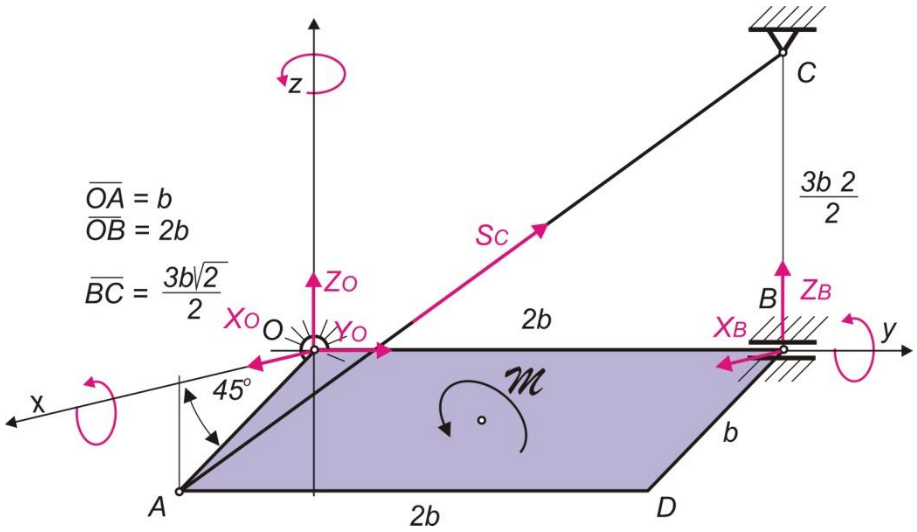
Primer 5.5

Homogena pravougaona ploča OABD, strana OB=2b, OA=b i težine G, oslonjena je u temenu O sfernim zglobom a u temenu B cilindričnim zglobom. Ploča je u nagnutom položaju prema ravni xOy održava lako uže AE. Nagibni ugao ploče je a=45o. Oslonac E se nalazi iznad B na udaljenosti $c = \frac{3b\sqrt{2}}{2}$. U ravni ploče deluje spreg $M = Gb\frac{\sqrt{2}}{3}$.

Naći reakcije oslonaca O i B kao i silu u užetu.



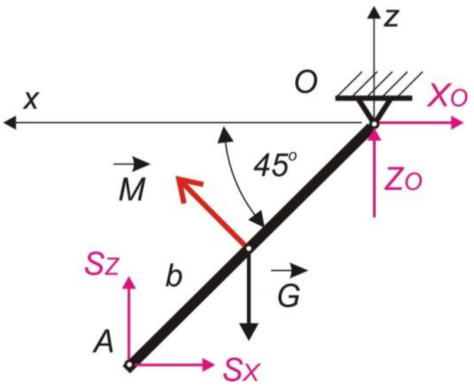
Telo osloboditi veza a njihov uticaj zameniti reakcijama veza



$\overrightarrow{F_O} = (X_0, Y_O, Z_O)$

$\overrightarrow{F_B} = (X_B, 0, Z_B)$

$\vec{G} = (0,0G)$



$\vec{M} = (M\cos 45°, 0, M\cos 45°) = \left(M\frac{\sqrt{2}}{2}, 0, M\frac{\sqrt{2}}{2}\right) = \left(\frac{b}{3}G, 0, \frac{b}{3}G\right)$

$\vec{S} = (S_X, S_Y, S_Z) = S \cdot \vec{s_o}$ a krajnje tačke štapa $C\left(0, 2b, \frac{3b\sqrt{2}}{2}\right)$ $A\left(b\frac{\sqrt{2}}{2}, 0, -b\frac{\sqrt{2}}{2} 0\right)$

$$\vec{s_o} = \overrightarrow{CA_0} = \frac{\overrightarrow{CA}}{|\overrightarrow{CA}|}$$

$\overrightarrow{CA} = (x_C - x_A)\cdot\vec{\imath} + (y_C - y_A)\cdot\vec{\jmath} + (z_C - z_A)\cdot\vec{k}$

$\overrightarrow{CA} = \left(-b\frac{\sqrt{2}}{2}\right)\cdot\vec{\imath} + 2b\cdot\vec{\jmath} + \left(\frac{3b\sqrt{2}}{2} + b\frac{\sqrt{2}}{2}\right)\cdot\vec{k} = -b\frac{\sqrt{2}}{2}\cdot\vec{\imath} + 2b\cdot\vec{\jmath} + 2b\sqrt{2}\cdot\vec{k}$

$|\overrightarrow{CA}| = \sqrt{\left(b\frac{\sqrt{2}}{2}\right)^2 + (2b)^2 + \left(2b\sqrt{2}\right)^2} = \frac{5\sqrt{2}}{2}b$

$\vec{s_o} = \overrightarrow{CA_0} = \frac{\overrightarrow{CA}}{|\overrightarrow{CA}|} == \frac{-b\frac{\sqrt{2}}{2}}{\frac{5\sqrt{2}}{2}b}\cdot\vec{\imath} + \frac{2b}{\frac{5\sqrt{2}}{2}b}\cdot\vec{\jmath} + \frac{2b\sqrt{2}}{\frac{5\sqrt{2}}{2}b}\cdot\vec{k} = -\frac{1}{5}\cdot\vec{\imath} + \frac{2\sqrt{2}}{5}\cdot\vec{\jmath} + \frac{4}{5}\cdot\vec{k}$

$\vec{S} = S\cdot\vec{s_o} = -\frac{S}{5}\cdot\vec{\imath} + \frac{2S\sqrt{2}}{5}\cdot\vec{\jmath} + \frac{4S}{5}\cdot\vec{k}$

$\overrightarrow{M_O^{\vec{S}}} = \vec{r_A}\times\vec{S} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{b\sqrt{2}}{2} & 0 & -\frac{b\sqrt{2}}{2} \\ -\frac{S}{5} & \frac{2S\sqrt{2}}{5} & \frac{4S}{5} \end{vmatrix} =$

$\overrightarrow{M_O^{\vec{S}}} = \frac{2S\sqrt{2}}{5}\cdot\frac{b\sqrt{2}}{2}\cdot\vec{\imath} - \left(\frac{b\sqrt{2}}{2}\cdot\frac{4S}{5} - \frac{S}{5}\cdot\frac{b\sqrt{2}}{2}\right)\cdot\vec{\jmath} + \frac{b\sqrt{2}}{2}\cdot\frac{2S\sqrt{2}}{5}\cdot\vec{k}$

$\overrightarrow{M_O^{\vec{S}}} = \frac{2Sb}{5}\cdot\vec{\imath} - \frac{3Sb\sqrt{2}}{10}\cdot\vec{\jmath} + \frac{2Sb}{5}\cdot\vec{k}$

$\vec{M} = (M\cos 45°, 0, M\cos 45°) = \left(M\frac{\sqrt{2}}{2}, 0, M\frac{\sqrt{2}}{2}\right) = \left(\frac{b}{3}G, 0, \frac{b}{3}G\right)$

1. $\sum X_i = X_O + X_B - S_X = 0$
2. $\sum Y_i = Y_O + S_Y = 0$
3. $\sum Z_i = Z_O + Z_B - G + \frac{4}{5}S = 0$

4. $\sum M_x = -G \cdot b + \frac{b}{3}G + \frac{2b}{5}S + Z_B \cdot 2a = 0$
5. $\sum M_y = -\frac{3b\sqrt{2}}{10}S + G\frac{b}{2}\frac{\sqrt{2}}{2} = 0$
6. $\sum M_z = \frac{2b}{5}S + G\frac{b}{3} + X_B \cdot 2b = 0$

Rešavanje sistema jednačina

5) $\rightarrow = S = G\frac{b}{2}\frac{\sqrt{2}}{2}\frac{10}{3b\sqrt{2}} = \frac{5}{6}G$

6)$\rightarrow X_B = \frac{1}{2b} \cdot \left(\frac{2b}{5}S + G\frac{b}{3}\right) = \frac{1}{2b} \cdot \left(\frac{2b}{5}\frac{5}{6}G + G\frac{b}{3}\right) = \frac{1}{3}G$

4) $\rightarrow Z_B = \frac{1}{2b}\left(-Gb + \frac{b}{3}G + \frac{2b}{5}\frac{5}{6}G\right) = \frac{1}{6}G$

3)$\rightarrow$ $Z_O = G - Z_B - \frac{4}{5}S = G - \frac{1}{6}G - \frac{4}{5}\frac{5}{6}G = \frac{1}{6}G$

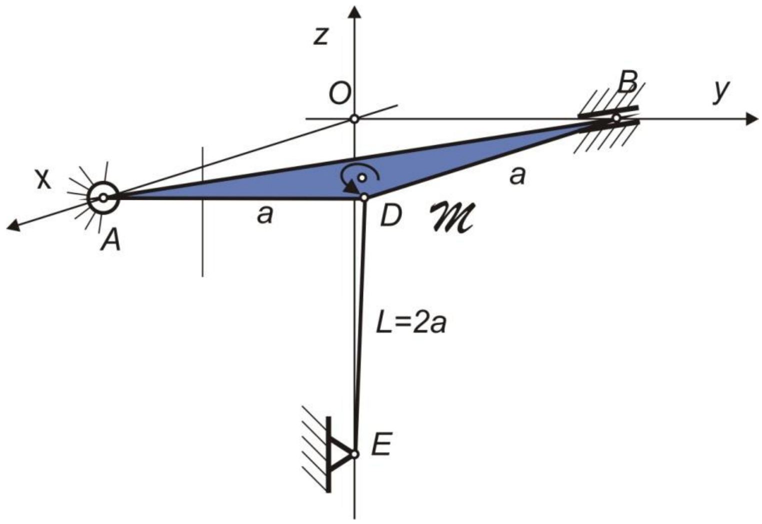
2) $\rightarrow Y_O = -S_Y = -\frac{2\sqrt{2}}{5}\frac{5}{6}G == -\frac{\sqrt{2}}{3}G$

1) $\rightarrow X_O = S_X - X_B = \frac{1}{5}\frac{5}{6}G - \frac{1}{3}G = -\frac{1}{6}G$

Primer 5.6

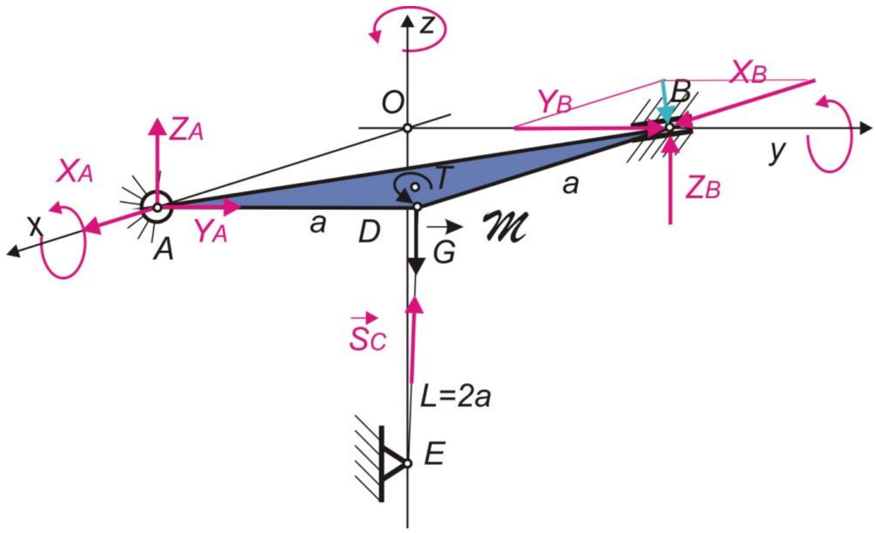
Homogena ploča oblika ravnokrakog pravouglog trougla vezana je za podlogu u tačkama A, sfernim ležištem i u B šarnirnim ležištem. Osa šarnirnog zgloba B je u pravcu hipotenuze trougle ploče. Ploču u horizontalnoj ravni održava kosi laki štap ED, dužine L=2a. Zglob E je na Oz osi. Ploča ima težinu G i katete AD=DB=a. U ravni ploče deluje spreg $M = Ga\sqrt{2}$.

Naći otpore oslonaca i silu u štapu.



Rešavanje počinje oslobađanjem veza, ali treba uočiti da je osa oslonca B u pravcu hipotenuze trougaone ploče, pa je normalan na nju. Komponenta u horizontalnoj ravni može se razložiti na dve , a pošto horizontalna reakcija F_B sa osama zaklapa isti ugao to su i projekcije iste.

$X_B = F_H\frac{\sqrt{2}}{2}$, odnosno $Y_B = F_H\frac{\sqrt{2}}{2}$, $\quad \overrightarrow{F_B} = \left(F_H\frac{\sqrt{2}}{2}, F_H\frac{\sqrt{2}}{2}, Z_B\right)$



$$\vec{G} = (0,0,-G)$$

$$\overrightarrow{F_A} = (X_A, Y_A, Z_A)$$

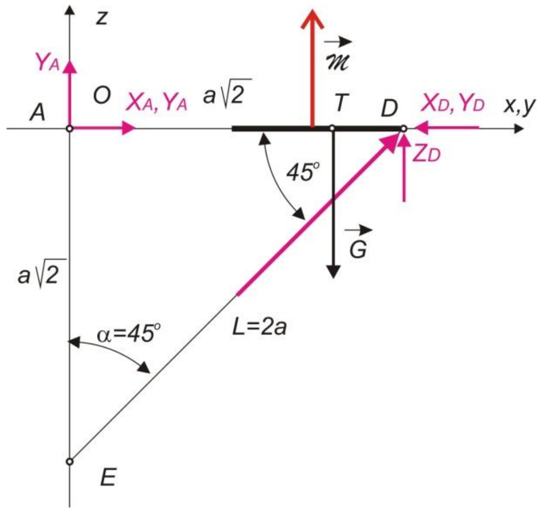
$$\vec{M} = (0,0,M)$$

$$\vec{M} = \left(0,0,\sqrt{2}Ga\right)$$

$$B(0,a,0)$$

$$A(a,0,0)$$

$$T\left(\frac{2}{3}a, \frac{2}{3}a, 0\right)$$



$$\overline{AD} = \sqrt{a^2 + a^2} = a\sqrt{2}$$

$$\overline{AE} = \sqrt{4a^2 - 2a^2} = a\sqrt{2}$$

$$E\left(0,0,-a\sqrt{2}\right)$$

$$D(a,a,0)$$

$$\vec{S} = (S_X, S_Y, S_Z) = S \cdot \overrightarrow{s_o}$$

$$\overrightarrow{s_o} = \overrightarrow{DE_0} = \frac{\overrightarrow{DE}}{|\overrightarrow{DE}|} = \frac{\overrightarrow{DE}}{2a} = \frac{1}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath} + \frac{\sqrt{2}}{2}\vec{k}$$

$$\vec{S} = \frac{S}{2}\vec{\imath} + \frac{S}{2}\vec{\jmath} + \frac{S\sqrt{2}}{2}\vec{k}$$

$$\overrightarrow{M_O^{\vec{S}}} = \overrightarrow{r_D} \times \vec{S} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a & a & o \\ \frac{S}{2} & \frac{S}{2} & \frac{S\sqrt{2}}{2} \end{vmatrix} = \frac{\sqrt{2}}{2}Sa \cdot \vec{\imath} - \frac{\sqrt{2}}{2}Sa \cdot \vec{\jmath}$$

$$\overrightarrow{M_O^{\overrightarrow{F_B}}} = \overrightarrow{r_B} \times \overrightarrow{F_B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & a & 0 \\ F_H\frac{\sqrt{2}}{2} & F_H\frac{\sqrt{2}}{2} & Z_B \end{vmatrix} = Z_B a \cdot \vec{\imath} - F_H\frac{\sqrt{2}}{2}a \cdot \vec{k}$$

$$\overrightarrow{M_O^{\overrightarrow{F_A}}} = \overrightarrow{r_A} \times \overrightarrow{F_A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a & 0 & 0 \\ X_B & Y_B & Z_B \end{vmatrix} = -Z_A a \cdot \vec{\jmath} - Y_A a \cdot \vec{k}$$

$$\overrightarrow{M_O^{\vec{G}}} = \overrightarrow{r_T} \times \overrightarrow{F_B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{2a}{3} & \frac{2a}{3} & 0 \\ 0 & 0 & -G \end{vmatrix} = -\frac{2}{3}Ga \cdot \vec{i} + \frac{2}{3}Ga \cdot \vec{j}$$

1. $\sum X_i = X_A + F_H\frac{\sqrt{2}}{2} + \frac{1}{2}S = 0$
2. $\sum Y_i = Y_A + F_H\frac{\sqrt{2}}{2} + \frac{1}{2}S = 0$
3. $\sum Z_i = Z_A + Z_B - G + \frac{\sqrt{2}}{2}S = 0$
4. $\sum M_x = \frac{\sqrt{2}}{2}S \cdot a + Z_B \cdot a - \frac{2}{3}G \cdot a = 0$
5. $\sum M_y = -\frac{\sqrt{2}}{2}S \cdot a - Z_A \cdot a + \frac{2}{3}G \cdot a = 0$
6. $\sum M_z = -\frac{\sqrt{2}}{2}F_H \cdot a + Y_A \cdot a + \sqrt{2}Ga = 0$

3)→ $\frac{\sqrt{2}}{2}S = G - Z_A - Z_B$

4) → $Ga - Z_A a - Z_B a + Z_B \cdot a - \frac{2}{3}G \cdot a = 0 \rightarrow Z_A = \frac{G}{3}$

5) → $-Ga + Z_A a + Z_B a - Z_A \cdot a + \frac{2}{3}G \cdot a = 0 \rightarrow Z_B = \frac{G}{3}$

3)→ $\frac{\sqrt{2}}{2}S = G - Z_A - Z_B = G - \frac{G}{6} - \frac{G}{6} \rightarrow S = \frac{\sqrt{2}}{3}G$

6)→ $Y_A = \frac{\sqrt{2}}{2}F_H - \sqrt{2}G$

2)→ $\frac{\sqrt{2}}{2}F_H - \sqrt{2}G + F_H\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{3}G = 0 \rightarrow F_H = \frac{5}{6}G$

1) → $X_A = -F_H\frac{\sqrt{2}}{2} - \frac{1}{2}S = -\frac{5\sqrt{2}}{12}G - \frac{1}{2}\frac{\sqrt{2}}{3}G = -\frac{7\sqrt{2}}{12}G$

$$X_B = F_H\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{5}{6}G = \frac{5\sqrt{2}}{12}G$$

$$Y_B = F_H\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \cdot \frac{5}{6}G = \frac{5\sqrt{2}}{12}G$$

$$F_B = \sqrt{\frac{25 \cdot 2G^2}{144} + \frac{25 \cdot 2G^2}{144} + \frac{G^2}{9}} = \frac{\sqrt{29}}{6}\ G$$