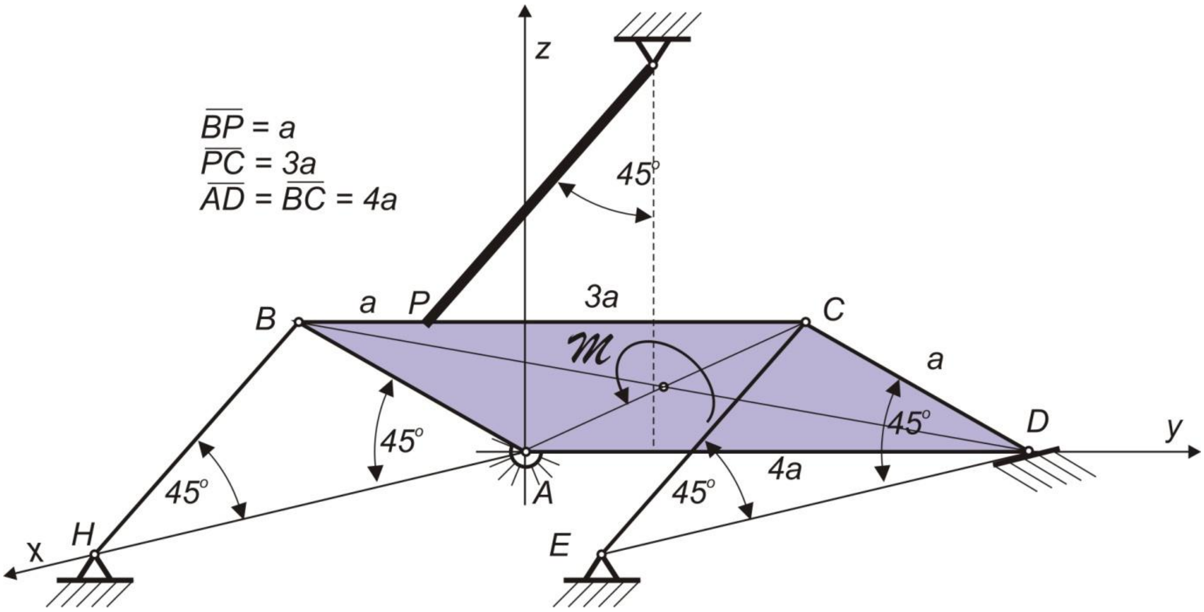
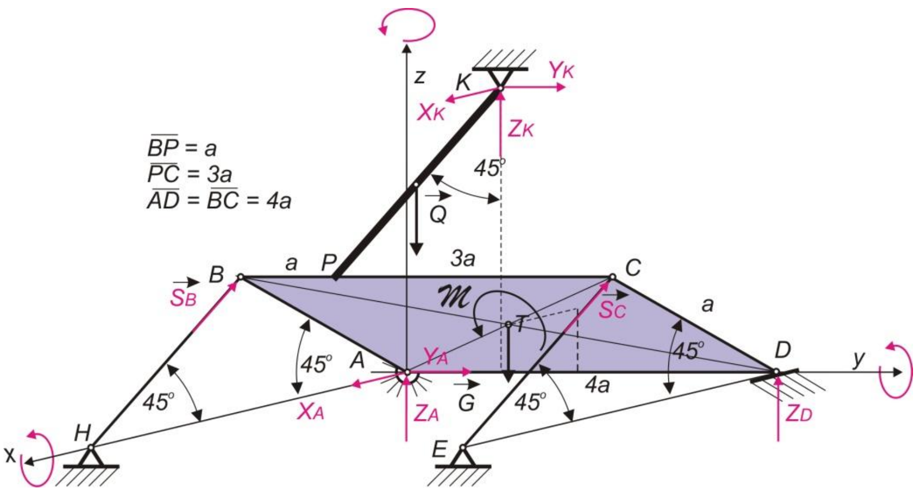
Primer 5.7

Pravougaona homogena ploča ABCD, težine G i stranica a i 4a, vezana je sfernim zglobom A za postolje, a u tački D slobodno oslonjena na horizontalnu glatku površinu. U prikazanom položaju na slici održavaju je laki štapovi BH i CE koji su upravni na ploču. Ploča sa horizontalnom ravni xAy zaklapa ugao od 45°. Na ploču se u tački P oslanja štap KP, težine Q, a zglobno je vezan u tački K. Tačka K nalazi se vertikalno iznad y ose, a ima koordinate $K(0, a, a\sqrt{2})$ i leži u ravni paralelnoj ravni xAz udaljenoj za a. Štap zaklapa sa vertikalom ugao od 45°. U ravni ploče deluje moment $\mathcal{M}$=Ga, sa smerom kao na slici.

Odrediti sile u svernom zglobu A, reakciju glatke površi D i sile u štapovima BH i CE. Poznato je $G = 10\ kN,\ Q = \frac{\sqrt{2}}{2}G = 14.1\ KN, a = 1m.$

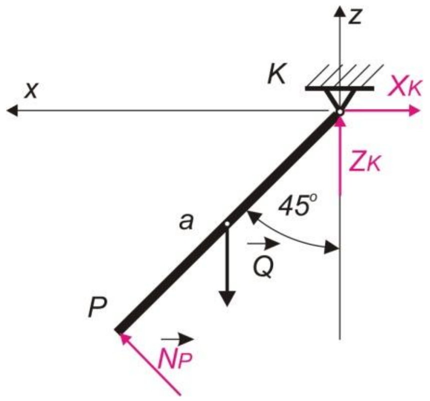


Analizom sistema tela zaključuje se da se mora izvršiti razdvajanje jer ima 8 nepoznatih, sile u štapovima imaju definisan pravac, reakcija glatke površi je vertikalna sila, teški štap je u ravni paralelnoj xAz pa nema komponente u pravcu Y ose, a šest uslova ravnoteže.



$$\vec{F_A} = (X_A, Y_A, Z_A)$$
$$\vec{F_D} = (0, 0, Z_D)$$
$$\vec{F_K} = (X_K, 0, Z_K)$$
$$\vec{S_B} = (S_{BX}, 0, S_{BZ})$$
$$\vec{S_C} = (S_{CX}, 0, S_{CZ})$$

Posebno se rešava teški štap zglobno vezan u tački K. Problem je ravanski pa se primenjuju tri poznata uslova ravnoteže. Pravac normalne reakcije ivice ploče je upravan na štap u tački oslanjanja.



$Q=\frac{\sqrt{2}}{2}G=14.1\ kN$

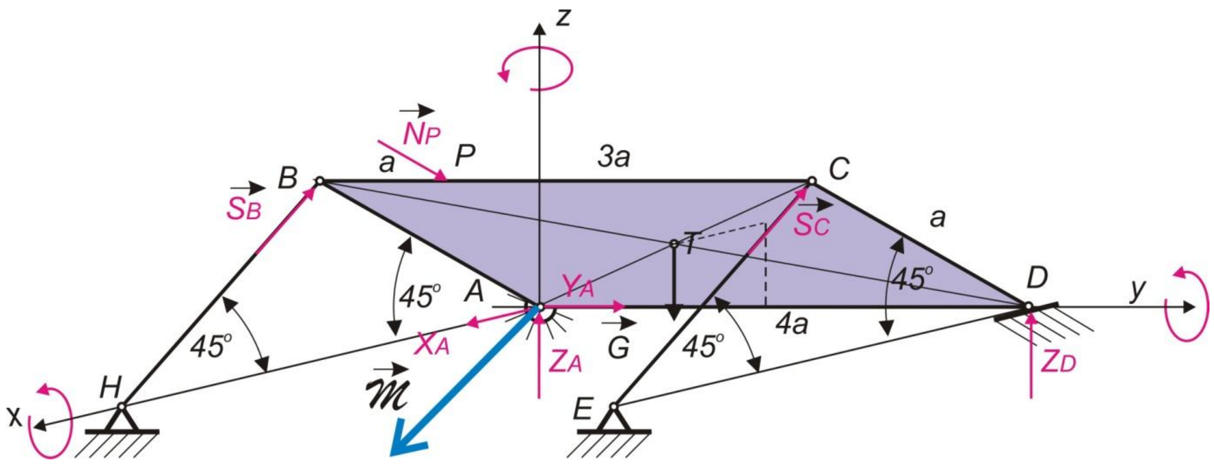
1. $\sum X_i=-X_K+N_P sin45°=0$
2. $\sum Z_i=Z_K+N_P cos45°-Q=0$
3. $\sum M_K=N_P\cdot a-Q\cdot\frac{a}{2}sin45°=0$

3) $\rightarrow N_P=Q\cdot\frac{1}{2}sin45°=\frac{\sqrt{2}}{2}G\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{G}{4}=2.5\ kN$

1) $\rightarrow X_K=N_P sin45°=\frac{G}{4}\cdot\frac{\sqrt{2}}{2}=\frac{G\sqrt{2}}{8}$

2) $\rightarrow Z_K=Q-N_P cos45°=\frac{\sqrt{2}}{2}G-\frac{G}{4}\cdot\frac{\sqrt{2}}{2}=\frac{3\sqrt{2}}{2}G$

Dobija se koje deluje na pčoču $\overrightarrow{N_P}=\left(-N_P\frac{\sqrt{2}}{2},0,-N_P\frac{\sqrt{2}}{2}\right)=\left(-\frac{\sqrt{2}}{8}G,0,-\frac{\sqrt{2}}{8}G\right)$



Težina G $\vec{G}=(0,0,-G)$

Muoment u ravni ploče ABCD

$\vec{M}=(M\cos 45°,0,-M\cos 45°)=\left(M\frac{\sqrt{2}}{2},0,-M\frac{\sqrt{2}}{2}\right)=\left(\frac{\sqrt{2}}{2}Ga,0,-\frac{\sqrt{2}}{2}Ga\right)$

Sila lakog štapa BH , sila u pravcu štapa BH, obeležena sa S_B.

$\overrightarrow{S_B}=(-S_B\cos 45°,0,S_B\cos 45°)=\left(-S_B\frac{\sqrt{2}}{2},0,S_B\frac{\sqrt{2}}{2}\right)$

Sila lakog štapa CE , sila u pravcu štapa CE, obeležena sa S_C.

$\overrightarrow{S_C}=(-S_C\cos 45°,0,S_C\cos 45°)=\left(-S_C\frac{\sqrt{2}}{2},0,S_C\frac{\sqrt{2}}{2}\right)$

$\overrightarrow{F_A}=(X_A,Y_A,Z_A)$

$\overrightarrow{F_D}=(0,0,Z_D)$

Uslovi ravnoteže: $\sum X_i=0$; $\sum Y_i=0$; $\sum Z_i=0$ $\sum M_{Xi}=0$; $\sum M_{Yi}=0$; $\sum M_{Zi}=0$

1. $\sum X_i = X_A - S_{BX} - S_{CX} - N_{PX} = 0$
2. $\sum Y_i = Y_A = 0$
3. $\sum Z_i = Z_A + Z_D + S_{BZ} + S_{CZ} - N_{PZ} - G = 0$
4. $\sum M_x = M_x - G \cdot 2a - N_{PZ} \cdot a + S_{CZ} \cdot 4a + Z_D \cdot 4a = 0$
5. $\sum M_y = -S_B \cdot a + G \cdot \frac{a\sqrt{2}}{2} - S_C \cdot a = 0$
6. $\sum M_z = -M_z + S_{CX} \cdot 4a + N_{PX} \cdot a = 0$

Rešavanje sistema jednačina

2) $\rightarrow Y_A = 0$

6)$\rightarrow S_{CX} = \frac{-M_z + N_{PX} \cdot a}{4a} = -\frac{1}{4a} Ga + \frac{\sqrt{2}}{8} G \cdot \frac{a}{4a} = \frac{3\sqrt{2}}{32} G$

$S_{CX} = S_C \cos 45° \rightarrow S_C = \frac{S_{CX}}{\cos 45°} = \frac{3\sqrt{2}}{32} G \cdot \frac{1}{\frac{\sqrt{2}}{2}} = \frac{3}{16} G = 1.875\ kN$

5) $\rightarrow S_B = \frac{1}{a}\left(G \cdot \frac{a\sqrt{2}}{2} - S_C \cdot a\right) = \left(G \cdot \frac{\sqrt{2}}{2} - \frac{3}{16} G\right) = \frac{8\sqrt{2}-3}{16} G = 5.196 kN$

4) $\rightarrow Z_D = \frac{1}{4a}(-M_x + G \cdot 2a + N_{PZ} \cdot a - S_{CZ} \cdot 4a) = \frac{1}{4a}\left(-Ga \cdot \frac{\sqrt{2}}{2} + G \cdot 2a + \frac{\sqrt{2}}{8} Ga - \frac{3\sqrt{2}}{32} G4a\right)$

$Z_D = \frac{8+2\sqrt{2}}{16} G = 6.767\ kN$

3)$\rightarrow Z_A = -Z_D - S_{BZ} - S_{CZ} + N_{PZ} + G = -\frac{8-3\sqrt{2}}{16} G - \frac{8\sqrt{2}-3}{16} G \frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{32} G + \frac{\sqrt{2}}{8} G + G$

$Z_A = \frac{5\sqrt{2}}{32} G = 2.209\ kN$

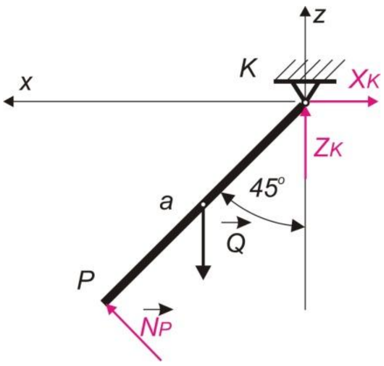
1) $X_A = S_{BX} + S_{CX} + N_{PX} = \frac{16-3\sqrt{2}}{32} G + \frac{3\sqrt{2}}{32} G + \frac{\sqrt{2}}{8} G = \frac{4+\sqrt{2}}{8} G = 6.767\ kN$

DRUGI NAČIN ODREĐIVANJA PRAVACA DEJSTVA SILA I MOMENATA

Prvi deo zadatka sa analizom broja nepoznatih i posebna analiza ravnoteže štapa PK vezanog u tački K, mora se primenuti kako bi se odredila sila Np. Potom se može na ploču uvesti sila normalne reakcije Np i rešavati. Ako bi se analizirao ceo sistem tela, tada se umesto reakcija veza zgloba K uvodi sa komponentama određenim iz analize ravnoteže štapa PK.

Posebno se rešava teški štap zglobno vezan u tački K. Problem je ravanski pa se primenjuju tri poznata uslova ravnoteže. Pravac normalne reakcije ivice ploče je upravan na štap u tački oslanjanja.

$Q = \frac{\sqrt{2}}{2} G = 14.1\ kN$



1. $\sum X_i = -X_K + N_P sin45° = 0$
2. $\sum Z_i = Z_K + N_P cos45° - Q = 0$
3. $\sum M_K = N_P \cdot a - Q \cdot \frac{a}{2} sin45° = 0$

3) $\rightarrow N_P = Q \cdot \frac{1}{2} sin45° = \frac{\sqrt{2}}{2} G \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{G}{4} = 2.5\ kN$

1) $\rightarrow X_K = N_P sin45° = \frac{G}{4} \cdot \frac{\sqrt{2}}{2} = \frac{G\sqrt{2}}{8}$

2) $\rightarrow Z_K = Q - N_P cos45° = \frac{\sqrt{2}}{2} G - \frac{G}{4} \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2} G$

Dobija se koje deluje na pčoču $\overrightarrow{N_P} = \left(-N_P \frac{\sqrt{2}}{2}, 0, -N_P \frac{\sqrt{2}}{2}\right) = \left(-\frac{\sqrt{2}}{8} G, 0, -\frac{\sqrt{2}}{8} G\right)$

$\overrightarrow{F_A} = (X_A, Y_A, Z_A)$

$\overrightarrow{F_D} = (0, 0, Z_D)$

$\overrightarrow{S_B} = (S_{BX}, 0, S_{BZ}) = S_B \cdot \overrightarrow{s_{Bo}}$ a krajnje tačke štapa $B\left(a\frac{\sqrt{2}}{2}, 0, a\frac{\sqrt{2}}{2}\right)$ $H\left(a\sqrt{2}, 0, 0\right)$

$\overrightarrow{S_C} = (S_{CX}, 0, S_{CZ}) = S_C \cdot \overrightarrow{s_{Co}}$ a krajnje tačke štapa $C\left(a\frac{\sqrt{2}}{2}, 4a, a\frac{\sqrt{2}}{2}\right)$ $E\left(a\sqrt{2}, 4a, 0\right)$

Kako su poznate krajnje tačke lakih štapova BH i CE to su vektori u njima jednaki proizvodu intenziteta i jediničnih vektora koji se određuju:

$$\overrightarrow{s_{Bo}} = \frac{\overrightarrow{BH}}{\left|\overrightarrow{BH}\right|}$$

$B\left(a\frac{\sqrt{2}}{2}, 0, a\frac{\sqrt{2}}{2}\right)$ $H\left(a\sqrt{2}, 0, 0\right)$

$\overrightarrow{BH} = (x_B - x_H) \cdot \vec{\imath} + (y_B - y_H) \cdot \vec{\jmath} + (z_B - z_H) \cdot \vec{k} = -a\frac{\sqrt{2}}{2} \cdot \vec{\imath} + a\frac{\sqrt{2}}{2}\vec{k}$

$\left|\overrightarrow{BH}\right| = \sqrt{\left(\frac{a\sqrt{2}}{2}\right)^2 + \left(\frac{a\sqrt{2}}{2}\right)^2} = a$

$\overrightarrow{s_{Bo}} = \frac{\overrightarrow{BH}}{\left|\overrightarrow{BH}\right|} = -\frac{\sqrt{2}}{2} \cdot \vec{\imath} + \frac{\sqrt{2}}{2}\vec{k}$

$\overrightarrow{S_B} = S_B \cdot \overrightarrow{s_{Bo}} = -S_B \cdot \frac{\sqrt{2}}{2} \cdot \vec{\imath} + S_B \cdot \frac{\sqrt{2}}{2}\vec{k} = \left(-S_B \cdot \frac{\sqrt{2}}{2}, 0, S_B \cdot \frac{\sqrt{2}}{2}\right)$

$$\overrightarrow{s_{Co}} = \frac{\overrightarrow{CE}}{\left|\overrightarrow{CE}\right|}$$

$C\left(a\frac{\sqrt{2}}{2}, 4a, a\frac{\sqrt{2}}{2}\right)$ $E\left(a\sqrt{2}, 4a, 0\right)$

$\overrightarrow{CE} = (x_C - x_E) \cdot \vec{\imath} + (y_C - y_E) \cdot \vec{\jmath} + (z_C - z_E) \cdot \vec{k} = -a\frac{\sqrt{2}}{2} \cdot \vec{\imath} + a\frac{\sqrt{2}}{2}\vec{k}$

$\left|\overrightarrow{CE}\right| = \sqrt{\left(\frac{a\sqrt{2}}{2}\right)^2 + \left(\frac{a\sqrt{2}}{2}\right)^2} = a$

$\overrightarrow{s_{Co}} = \frac{\overrightarrow{CE}}{|\overrightarrow{CE}|} = -\frac{\sqrt{2}}{2} \cdot \vec{i} + \frac{\sqrt{2}}{2}\vec{k}$

$\overrightarrow{S_C} = S_C \cdot \overrightarrow{s_{Co}} = -S_C \cdot \frac{\sqrt{2}}{2} \cdot \vec{i} + S_C \cdot \frac{\sqrt{2}}{2}\vec{k} = \left(-S_C \cdot \frac{\sqrt{2}}{2}, 0, S_C \cdot \frac{\sqrt{2}}{2}\right)$

Moment u ravni ploče

$\vec{M} = (M\cos 45°, 0, -M\cos 45°) = \left(M\frac{\sqrt{2}}{2}, 0, -M\frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}Ga, 0, -\frac{\sqrt{2}}{2}Ga\right)$

Sile prave momente za tačku A koji su

$$\overrightarrow{M_A^{\vec{G}}} = \overrightarrow{r_T} \times \vec{G} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{a\sqrt{2}}{4} & 2a & \frac{a\sqrt{2}}{4} \\ 0 & 0 & -G \end{vmatrix} = -2aG \cdot \vec{i} + \frac{a\sqrt{2}}{4}G \cdot \vec{j}$$

$$\overrightarrow{M_A^{\overrightarrow{N_P}}} = \overrightarrow{r_P} \times \overrightarrow{N_P} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a\sqrt{2} & a & a\sqrt{2} \\ -\frac{\sqrt{2}}{8}G & 0 & -\frac{\sqrt{2}}{8}G \end{vmatrix} = -\frac{a\sqrt{2}}{8}G\vec{i} - \frac{G\sqrt{2}}{8}a \cdot \vec{k}$$

$$\overrightarrow{M_A^{F_K}} = \frac{3a\sqrt{2}}{2}G \cdot \vec{i} - \frac{Ga}{4} \cdot \vec{j} - \frac{Ga\sqrt{2}}{8} \cdot \vec{k}$$

$$\overrightarrow{M_A^{\overrightarrow{S_B}}} = \overrightarrow{r_B} \times \overrightarrow{S_B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{a\sqrt{2}}{2} & 0 & \frac{a\sqrt{2}}{2} \\ -S_B \cdot \frac{\sqrt{2}}{2} & 0 & S_B \cdot \frac{\sqrt{2}}{2} \end{vmatrix} = \left(\frac{a\sqrt{2}}{2}S_B \cdot \frac{\sqrt{2}}{2} + \frac{a\sqrt{2}}{2}S_B \cdot \frac{\sqrt{2}}{2}\right)\vec{j} = S_B \cdot a \cdot \vec{j}$$

$$\overrightarrow{M_A^{\overrightarrow{S_C}}} = \overrightarrow{r_C} \times \overrightarrow{S_C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{a\sqrt{2}}{2} & 4a & \frac{a\sqrt{2}}{2} \\ -S_C \cdot \frac{\sqrt{2}}{2} & 0 & S_C \cdot \frac{\sqrt{2}}{2} \end{vmatrix} =$$

$$\overrightarrow{M_A^{\overrightarrow{S_C}}} = 4a \cdot S_C \cdot \frac{\sqrt{2}}{2} \cdot \vec{i} + \left(\frac{a\sqrt{2}}{2}S_C \cdot \frac{\sqrt{2}}{2} + \frac{a\sqrt{2}}{2}S_C \cdot \frac{\sqrt{2}}{2}\right)\vec{j} - 4a \cdot S_C \cdot \frac{\sqrt{2}}{2} \cdot \vec{k}$$

$$\overrightarrow{M_A^{\overrightarrow{S_C}}} = 2a\sqrt{2} \cdot S_C \cdot \vec{i} + S_C \cdot a\vec{j} - 2a\sqrt{2} \cdot S_C \cdot \vec{k}$$

$$\overrightarrow{M_A^{\overrightarrow{F_D}}} = \overrightarrow{r_D} \times \overrightarrow{F_D} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4a & 0 \\ 0 & 0 & Z_D \end{vmatrix} = 4aZ_D \cdot \vec{i}$$

1. $\sum X_i = X_A - S_{BX} - S_{CX} - N_{PX} = 0$
2. $\sum Y_i = Y_A = 0$
3. $\sum Z_i = Z_A + Z_D + S_{BZ} + S_{CZ} - N_{PZ} - G = 0$

4. $\sum M_x = M_x - G \cdot 2a - N_{PZ} \cdot a + S_{CZ} \cdot 4a + Z_D \cdot 4a = 0$

5. $\sum M_y = -S_B \cdot a + G \cdot \frac{a\sqrt{2}}{2} - S_C \cdot a = 0$

6. $\sum M_z = -M_z + S_{CX} \cdot 4a + N_{PX} \cdot a = 0$